

Evaluating Determinants

$$1. \begin{vmatrix} 3 & 9 & 6 \\ 4 & -4 & 4 \\ 10 & 5 & 20 \end{vmatrix} = -300$$

$$2. \begin{vmatrix} 25 & 40 & 15 \\ -3 & 6 & 21 \\ 8 & 12 & 20 \end{vmatrix} = 4560$$

$$3. \begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix} = 132$$

$$4. \begin{vmatrix} 28 & 27 & 25 \\ 31 & 30 & 26 \\ 36 & 35 & 30 \end{vmatrix} = 7$$

$$5. \begin{vmatrix} 26 & 29 & 29 \\ 25 & 30 & 27 \\ 25 & 28 & 26 \end{vmatrix} = -101$$

$$6. \begin{vmatrix} 35 & 73 & 16 \\ 38 & 80 & 23 \\ 32 & 67 & 16 \end{vmatrix} = -15$$

$$7. \begin{vmatrix} 29 & 30 & 33 \\ 35 & 38 & 42 \\ 28 & 29 & 32 \end{vmatrix} = 5$$

$$8. \begin{vmatrix} 13 & 16 & 19 \\ 27 & 33 & 39 \\ 28 & 34 & 40 \end{vmatrix} = 0$$

$$9. \begin{vmatrix} 22 & 32 & 27 \\ 27 & 41 & 34 \\ 20 & 30 & 25 \end{vmatrix} = 0$$

$$10. \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix} = 1$$

$$11. \begin{vmatrix} 30 & 11 & 20 & 38 \\ 12 & 6 & 0 & 18 \\ 11 & -2 & 36 & 3 \\ 19 & 6 & 17 & 22 \end{vmatrix} = 18$$

$$12. \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 5$$

Solve using Cramer's rule:

$$1. \quad x + y + z = 6$$

$$2x - y - z = -3$$

$$x - 3y + 2z = 1$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 1 & -3 & 2 \end{vmatrix} \Rightarrow D = -15, D_x = \begin{vmatrix} 6 & 1 & 1 \\ -3 & -1 & -1 \\ 1 & -3 & 2 \end{vmatrix} \Rightarrow D_x = -15, D_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & -3 & -1 \\ 1 & 1 & 2 \end{vmatrix} \Rightarrow D_y = -30,$$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & -1 & -3 \\ 1 & -3 & 1 \end{vmatrix} \Rightarrow D_z = -45, \frac{D_x}{D} = \frac{-15}{-15} = 1, \frac{D_y}{D} = \frac{-30}{-15} = 2, \frac{D_z}{D} = \frac{-45}{-15} = 3 \Rightarrow \{(1, 2, 3)\}$$

$$2. \quad x - y + z = 3$$

$$3x + 2y - z = 1$$

$$4x - 2y - 3z = -2$$

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 3 & 2 & -1 \\ 4 & -2 & -3 \end{vmatrix} \Rightarrow D = -27, D_x = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & -1 \\ -2 & -2 & -3 \end{vmatrix} \Rightarrow D_x = -27, D_y = \begin{vmatrix} 1 & 3 & 1 \\ 3 & 1 & -1 \\ 4 & -2 & -3 \end{vmatrix} \Rightarrow D_y = 0$$

$$D_z = \begin{vmatrix} 1 & -1 & 3 \\ 3 & 2 & 1 \\ 4 & -2 & -2 \end{vmatrix} \Rightarrow D_z = -54, \frac{D_x}{D} = \frac{-27}{-27} = 1, \frac{D_y}{D} = \frac{0}{-27} = 0, \frac{D_z}{D} = \frac{-54}{-27} = 2, \{(1, 0, 2)\}$$

$$3. \quad x - 2y + 3z = 6$$

$$2x + y - z = 4$$

$$3x - y + 2z = -1$$

$$D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 2 \end{vmatrix} \Rightarrow D = 0, D_x = \begin{vmatrix} 6 & -2 & 3 \\ 4 & 1 & -1 \\ -1 & -1 & 2 \end{vmatrix} \Rightarrow D_x = 11, D_y = \begin{vmatrix} 1 & 6 & 3 \\ 2 & 4 & -1 \\ 3 & -1 & 2 \end{vmatrix} \Rightarrow D_y = -77,$$

$$D_z = \begin{vmatrix} 1 & -2 & 6 \\ 2 & 1 & 4 \\ 3 & -1 & -1 \end{vmatrix} \Rightarrow D_z = -55, \frac{D_x}{D} = \frac{11}{0} = \emptyset, \frac{D_y}{D} = \frac{-77}{0} = \emptyset, \frac{D_z}{D} = \frac{-55}{0} = \emptyset,$$

$$4. \quad 3x + 2y - 5z = 4$$

$$-x + y - z = 11$$

$$3x - y + 2z = -1$$

$$D = \begin{vmatrix} 3 & 2 & -5 \\ -1 & 1 & -1 \\ 3 & -1 & 2 \end{vmatrix} \Rightarrow D = 11, D_x = \begin{vmatrix} 4 & 2 & -5 \\ 11 & 1 & -1 \\ -1 & -1 & 2 \end{vmatrix} \Rightarrow D_x = 12, D_y = \begin{vmatrix} 3 & 4 & -5 \\ -1 & 11 & -1 \\ 3 & -1 & 2 \end{vmatrix} \Rightarrow D_y = 219$$

$$D_z = \begin{vmatrix} 3 & 2 & 4 \\ -1 & 1 & 11 \\ 3 & -1 & -1 \end{vmatrix} \Rightarrow D_z = 86, \frac{D_x}{D} = \frac{12}{11}, \frac{D_y}{D} = \frac{219}{11}, \frac{D_z}{D} = \frac{86}{11} \left\{ \left(\frac{12}{11}, \frac{219}{11}, \frac{86}{11} \right) \right\}$$