Integration

Simplify each of the following:

1. $\int x^4 dx$
2. $\int e^x dx$
3. $\int \sin x dx$
4. $\int 5^x dx$
5. $\int \cos x dx$
6. $\int \frac{1}{x} dx$
7. $\int x^{-5} dx$
8. $\int \frac{2}{x+1} dx$
9. $\int \cos 3x dx$
10. $\int 7^{3x} dx$
11. $\int x^2 (x-3) dx$
12. $\int \frac{1}{x^5} d$
13. $\int x^4 - \frac{2}{x^3} dx$
14. $\int 2x^3 - 5x + 1 dx$

Related rates:

1. A point moves on a hyperbola $y^2 - 2x^2 = 2$, where x and y are differentiable functions of time. What is the rate of change of y with respect to time when $\frac{dx}{dt} = 1/2, x = 1, y = -2$
2. Water is flowing at a rate of 5ft3/min into a tank in the from of a cone of altitude 20 ft. and a base radius 10 ft. and with its vertex in the downward direction. How fast is the water rising when the water is 8ft. deep?
3. A balloon is being inflated at the rate of 15ft3/min. At what rate is the diameter increasing after five minutes.
4. A ladder 20 feet long leans against a vertical wall of house. The lower end slips away from the house at a rate of 3ft/sec. When the lower end is 10 ft from the wall, find the rate at which the ladder slides down the wall.
5. One ship leaves port and steams due north at 10 knots. Three hours later another ship leaves the same port and steams due west at 30 knots. How fast is the distance between then increasing when the first ship has been out of port for 5 hours.

Max/Min Problems:

1. In designing pages for a book, a publisher decides that the rectangular printed region on each page must have area 150cm^2. if the page must have 2.5 cm margins on each side and 3.75 cm margins top and bottom, find the dimensions of the page of the smallest possible area.
2. A rectangular field is to bee fenced on three sides with 2000m of fencing (the fourth side being adjacent to a fenced airport runway). Find the dimensions of the field in order that the area be the largest possible.
3. A box is made from a rectangular cardboard 20 cm by 36 cm by cutting equal squares from each of the corners and folding up the four sides. Determine the size of the squares that should be cut from each corner to produce a box with maximum volume.
4. Pop cans to hold 350 mL are made in the shape of a right cylinder. Find the dimensions of the can that will maximize its surface area. $\left(V = \pi r^2 h\right)$