Infinite Geometric Series:

1. Find the missing term:

a) a = 5 , r = 1/3 $S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{5}{1-\frac{1}{3}} \Rightarrow S_n = \frac{5}{\frac{2}{3}} \Rightarrow S_n = \frac{15}{2}$

b) a = -2, r = 2/3 $S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{-2}{1-\frac{2}{3}} \Rightarrow S_n = \frac{-2}{\frac{1}{3}} \Rightarrow S_n = -6$

c) a = 7, Sn = 11 $S_n = \frac{a}{1-r} \Rightarrow 11 = \frac{7}{1-r} \Rightarrow 11 - 11r = 7 \Rightarrow -11r = -4 \Rightarrow r = \frac{4}{11}$

d) a = -4/5, Sn = 20

$$S_n = \frac{a}{1-r} \Rightarrow 20 = \frac{-\frac{4}{5}}{1-r} \Rightarrow 20 - 20r = -\frac{4}{5} \Rightarrow 100 - 100r = -4 \Rightarrow -100r = -104 \Rightarrow r = \frac{104}{100} = \frac{26}{25}$$

2. Find the sum:

a) $\sum_{j=1}^{\infty} -2(1/3)^{j-1}$

$a = -2, r = \frac{1}{3}$

$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{-2}{1-\frac{1}{3}} \Rightarrow S_n = \frac{-2}{\frac{2}{3}} \Rightarrow S_n = -3$

b) $\sum_{j=1}^{\infty} 5(2/3)^{j-1}$

$a = 5, r = \frac{2}{3}$

$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{5}{1-\frac{2}{3}} \Rightarrow S_n = \frac{5}{\frac{1}{3}} \Rightarrow S_n = 15$

c) $\sum_{m=1}^{\infty} 9(1/4)^{m-1}$

$a = 9, r = \frac{1}{4}$

$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{9}{1-\frac{1}{4}} \Rightarrow S_n = \frac{9}{\frac{3}{4}} \Rightarrow S_n = 12$

3. a) A rubber ball dropped from a height of 34 meters rebounded on each bounce 5/8 of the height from which it fell. How far did it travel:
   i) in the 6^{th} bounce
   ii) in 10 bounces
   iii) in coming to rest

$$a = {}^{221}\!/_{4} = 55.25, r = {}^{5}\!/_{8}$$

$$a)\ l = ar^{n-1} \Rightarrow l = {}^{221}\!/_{4} \cdot \left({}^{5}\!/_{8}\right)^{6-1} \Rightarrow l = {}^{221}\!/_{4} \cdot \left({}^{5}\!/_{8}\right)^{5} \Rightarrow l = {}^{221}\!/_{4} \cdot \left({}^{3125}\!/_{32768}\right) \cong 5.269$$

$$b)\ S_n = \frac{a - rl}{(1-r)} \Rightarrow S_n = \frac{55.25 - {}^{5}\!/_{8} \cdot 5.269}{(1 - 5/8)} \Rightarrow S_n = \frac{55.25 - 3.293}{{}^{3}\!/_{8}} \Rightarrow S_n = \frac{51.957}{{}^{3}\!/_{8}} = 138.552$$

$$c)\ S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{{}^{221}\!/_{4}}{1 - {}^{5}\!/_{8}} \Rightarrow S_n = \frac{{}^{221}\!/_{4}}{{}^{3}\!/_{8}} \Rightarrow S_n = \frac{221 \cdot 8}{3 \cdot 4} = 147.33$$

b) Air resistance causes the path of each swing (after the first) of a pendulum bob to be 0.98 as long as that of the preceding swing. If the path of the first swing is 15 centimeters long, find how far did it travel:

i) in the 5th swing
ii) in 8 swings
iii) in coming to rest

$$a = 29.7, r = .98$$

$$a)\ l = ar^{n-1} \Rightarrow l = 29.7 \cdot (.98)^{5-1} \Rightarrow l = 29.7 \cdot (.98)^{4} \Rightarrow l = 29.7 \cdot (.9223) \cong 27.394$$

$$b)\ S_n = \frac{a - rl}{(1-r)} \Rightarrow S_n = \frac{29.7 - .98 \cdot (27.396)}{(1 - .98)} \Rightarrow S_n = \frac{29.7 - 26.848}{.02} \Rightarrow S_n = \frac{2.8519}{.02} = 142.596$$

$$c)\ S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{29.7}{1 - .98} \Rightarrow S_n = \frac{29.7}{.02} \Rightarrow S_n = 1485$$

4. Convert the following repeating decimals to a fraction:

a) $0.\overline{371}$

$$a = {}^{371}\!/_{1000}, r = {}^{1}\!/_{1000}$$

$$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{{}^{371}\!/_{1000}}{1 - {}^{1}\!/_{1000}} \Rightarrow S_n = \frac{{}^{371}\!/_{1000}}{{}^{999}\!/_{1000}} \Rightarrow S_n = \frac{371}{999}$$

b) $0.2\overline{6832}$

$$a = {}^{6832}\!/_{100000}, r = {}^{1}\!/_{10000}$$

$$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{{}^{6832}\!/_{100000}}{1 - {}^{1}\!/_{10000}} \Rightarrow S_n = \frac{{}^{6832}\!/_{100000}}{{}^{9999}\!/_{10000}} \Rightarrow S_n = \frac{6832}{99990}$$

$$.2 + \frac{6832}{99990} = \frac{2}{10} + \frac{6832}{99990} = \frac{199980 + 68320}{99990} = \frac{268300}{99990}$$

c) $4.\overline{57}$

$$a = {}^{57}\!/_{100}, r = {}^{1}\!/_{100}$$

$$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{{}^{57}\!/_{100}}{1 - {}^{1}\!/_{100}} \Rightarrow S_n = \frac{{}^{57}\!/_{100}}{{}^{99}\!/_{100}} \Rightarrow S_n = \frac{57}{99}$$

$$4 + \frac{57}{99} = \frac{396 + 57}{99} = \frac{453}{99}$$

General Problems:

1. Write each series in summation notation:

   a) (–21) + 7 + (-7/3) + 7/9 + … to 12 terms $\sum_{n=1}^{12} -21 \cdot \left(-\frac{1}{3}\right)^{n-1}$

   b) 3 + (-6) + (12) + (-24) + …. To 20 terms $\sum_{n=1}^{20} 3(-2)^{n-1}$

2. Expand and find the sum:

   a) $\sum_{n=1}^{7} 4(-3)^{n-1}$

   $a = 4, r = -3$

   $4, -12, 36, -108, 324, -972, 2916$

   $S_n = \frac{a - rl}{1 - r} \Rightarrow S_n = \frac{4 - (-3)(2916)}{1 - (-3)} \Rightarrow S_n = \frac{4 + 8748}{4} = \frac{8752}{4} = 2188$

   b) $\sum_{n=4}^{9} -2(3)^{n-1}$

   $a = -54, r = 3$

   $-54, -162, -486, -1458, -4374, -13122$

   $S_n = \frac{a - rl}{1 - r} \Rightarrow S_n = \frac{-54 - (-3)(13122)}{1 - (3)} \Rightarrow S_n = \frac{-54 + 39366}{-2} = \frac{39312}{-2} = 19656$