Geometric Sequences and Series

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1. Write the first four terms of the following G.S.
   a = -5, r = -1/4 __**-5**___, __**5/4**___, __**-5/16**___, __**5/64**___

2. What is the value of the sixth term if a = 5 and r = 2/3?

$$l = ar^{n-1} \Rightarrow l = 5\left(\tfrac{2}{3}\right)^{6-1} \Rightarrow l = 5\left(\tfrac{2}{3}\right)^{5} \Rightarrow l = 5\left(\tfrac{32}{243}\right) \Rightarrow l = \tfrac{160}{243}$$

3. Which term of the geometric sequence –20, -20/6, -20/36, ….to –20/46656?

$$l = ar^{n-1} \Rightarrow -\tfrac{20}{46656} = -20\left(\tfrac{1}{6}\right)^{n-1} \Rightarrow \tfrac{1}{46656} = \left(\tfrac{1}{6}\right)^{n-1} \Rightarrow \left(\tfrac{1}{6}\right)^{6} = \left(\tfrac{1}{6}\right)^{n-1}$$
$$6 = n-1 \Rightarrow n = 7$$

4. Insert three geometric means between 6 and 18750.

$$a = 6, l = 18750, n = 5$$
$$l = ar^{n-1} \Rightarrow 18750 = 6r^{5-1} \Rightarrow 3125 = r^4 \Rightarrow 5^5 = r^4 \Rightarrow \sqrt[4]{5^5} = \sqrt[4]{r^4} \Rightarrow r = 5\sqrt[4]{5}$$
$$6, \underline{6\cdot 5\sqrt[4]{5} = 30\sqrt[4]{5}}, \underline{30\sqrt[4]{5}\cdot 5\sqrt[4]{5} = 150\sqrt[4]{5^2}}, \underline{150\sqrt[4]{5^2}\cdot 5\sqrt[4]{5} = 750\sqrt[4]{5^3}}, 18750$$

5. Find the sum of the following geometric series:
   a) a = 2, r = -3, n = 7

$$S_n = \frac{a(1-r^n)}{(1-r)} \Rightarrow S_n = \frac{2(1-(-3)^7)}{(1-(-3))} \Rightarrow S_n = \frac{2(1-(-2187))}{4} \Rightarrow S_n = \frac{(2188)}{2} = 1094$$

   b) -5, 15, -45 ... to 8 terms

$$a = -5, n = 8, r = -3$$
$$S_n = \frac{a\left(1-r^n\right)}{(1-r)} \Rightarrow S_n = \frac{-5\left(1-(-3)^8\right)}{(1-(-3))} \Rightarrow S_n = \frac{-5(1-(6561))}{4} \Rightarrow S_n = \frac{-5(6560)}{4} = -8200$$

5. Find the missing terms a = 4, r = 4, S_n = 5460

Determine "l" and "n"

$$S_n = \frac{a-rl}{1-r} \Rightarrow 5460 = \frac{4-4l}{1-(4)} \Rightarrow 5460 = \frac{4-4l}{-3} \Rightarrow -16380 = 4-4l \Rightarrow$$
$$-16384 = -4l \Rightarrow l = 4096$$
$$l = ar^{n-1} \Rightarrow 4096 = 4(4)^{n-1} \Rightarrow 1024 = (4)^{n-1} \Rightarrow (4)^5 = (4)^{n-1} \Rightarrow 5 = n-1 \Rightarrow n$$

7. Expand and find the sum:

$$\sum_{n=6}^{9} -2(4)^{n-1}$$

Mini Sequence

$a = -2048$ (must determine value of 6th term), $l = ar^{n-1} \Rightarrow l = (-2)(4)^{6-1}$

$n = 4$ (remember starting on 6th term to the 9th term), $r = 4$

$-2048, -8192, -32768, -131072$

$$S_n = \frac{a - rl}{1 - r} \Rightarrow S_n = \frac{-2048 - (4)(-131072)}{1 - (4)} \Rightarrow S_n = \frac{-2048 + 524288}{-3} \Rightarrow S_n = \frac{522240}{-3} = -174080$$

8. Write in summation notation: 4 + 20 + 100 + 500 + …. to 24 terms

Determine "a" and "r"

$$\sum_{n=1}^{24} 4(5)^{n-1}$$

9. Write as a fraction: $0.\overline{459}$

$a = {}^{459}\!/_{1000}, r = {}^{1}\!/_{1000}$

$$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{459/1000}{1 - 1/1000} \Rightarrow S_n = \frac{459/1000}{99/1000} \Rightarrow S_n = \frac{459 \cdot 1000}{999 \cdot 1000} \Rightarrow S_n = \frac{459}{999}$$

10. If a ball is dropped from a height of 200 meters and rebounds 2/5 the distance it fell, find:

a) the distance it fell in the 4th bounce

$a = 200m + 200m \cdot {}^{2}\!/_{5} = 280m$

$$l = ar^{n-1} \Rightarrow l = 280m \cdot \left({}^{2}\!/_{5}\right)^{4-1} \Rightarrow l = 280m \cdot \left({}^{2}\!/_{5}\right)^{3} \Rightarrow l = 280m \cdot \left({}^{8}\!/_{125}\right) \Rightarrow l = {}^{448}\!/_{25}$$

b) the distance it fell in four bounces

$$Sn = \frac{a - rl}{1 - r} \Rightarrow Sn = \frac{280m - \left({}^{2}\!/_{5}\right)\left({}^{448}\!/_{25}\,m\right)}{1 - \left({}^{2}\!/_{5}\right)} \Rightarrow Sn = \frac{280m - \left({}^{2896}\!/_{125}\,m\right)}{\left({}^{3}\!/_{5}\right)} \Rightarrow$$

$$S_n = \frac{(35000m - 2896m)/125}{\left({}^{3}\!/_{5}\right)} \Rightarrow S_n = \frac{(32104m)/125}{\left({}^{3}\!/_{5}\right)} \Rightarrow S_n = \frac{32104m \cdot 5}{3 \cdot 125} = \frac{32104m}{75}$$

c) the distance it travels in coming to rest

$$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{280m}{1 - \left({}^{2}\!/_{5}\right)} \Rightarrow S_n = \frac{280m}{\left({}^{3}\!/_{5}\right)} \Rightarrow S_n = \frac{280m \cdot 5}{3} = \frac{1400m}{3}$$

11. Expand the find the sum $\sum_{n=1}^{\infty} 4\left(\frac{1}{3}\right)^{n-1}$

$4, \frac{4}{3}, \frac{4}{9}, \frac{4}{27}, \ldots$

$$S_n = \frac{a}{1-r} \Rightarrow S_n = \frac{4}{1-\left(\frac{1}{3}\right)} \Rightarrow S_n = \frac{4}{\left(\frac{2}{3}\right)} \Rightarrow S_n = \frac{4\cdot 3}{2} \Rightarrow S_n = 6$$

12. Find the 1st and 9th term if the 3rd term is 20 and the 7th term is 320.

Mini Sequence used to find "r"

$a = 20, l = 320, n = 5$

$l = ar^{n-1} \Rightarrow 320 = 20r^{5-1} \Rightarrow 16 = r^4 \Rightarrow 2^4 = r^4 \Rightarrow r = \pm 2$

Value of the 1st term: $l = ar^{n-1} \Rightarrow 320 = a(\pm 2)^{7-1} \Rightarrow 320 = a(\pm 2)^6 \Rightarrow 320 = a(64) \Rightarrow a = 5$

Value of the 9th term: $l = ar^{n-1} \Rightarrow l = 5(\pm 2)^{9-1} \Rightarrow l = 5(\pm 2)^8 \Rightarrow l = 5(256) \Rightarrow l = 1280$

13. The sum of a GP of eight terms if the 2rd term is -384 and the 7th term is 12

Mini Sequence

$a = -384, l = 12, n = 6$

$$l = ar^{n-1} \Rightarrow 12 = -384r^{6-1} \Rightarrow \frac{12}{-384} = r^5 \Rightarrow \frac{1}{-32} = r^5 \Rightarrow \left(-\frac{1}{2}\right)^5 = r^5 \Rightarrow \sqrt[5]{\left(-\frac{1}{2}\right)^5} = \sqrt[5]{r^5} \Rightarrow r = -\frac{1}{2}$$

Full Sequence

$$l = ar^{n-1} \Rightarrow 12 = a\left(-\frac{1}{2}\right)^{7-1} \Rightarrow 12 = a\left(-\frac{1}{2}\right)^6 \Rightarrow 12 = a\left(\frac{1}{64}\right) \Rightarrow a = 768$$

$$S_n = \frac{a - rl}{1-r} \Rightarrow S_n = \frac{768 - \left(-\frac{1}{2}\right)(12)}{1-\left(-\frac{1}{2}\right)} \Rightarrow S_n = \frac{768+6}{\frac{3}{2}} \Rightarrow S_n = \frac{774}{\frac{3}{2}} = \frac{774\cdot 2}{3} = 516$$

Arithmetic Sequences and Series

1. Which of the following sequences are arithmetic?
   a) 15,11,7,3… **yes** c) -3.5,-2,-0.5,1… **yes** c) 4,8,16,32… **no**

2. State the next 3 terms of each arithmetic sequence.
   a) 8, -1, -10… **-19, -28, -37** c) 1.25, 3.75, 6.25… **8.75, 11.25, 13.75**

3. Find the specified term for each arithmetic sequence.
   a) 25, 31, 37…; find t_{14}

$l = a + (n-1)d \Rightarrow l = 25 + (14-1)6 \Rightarrow l = 25 + (13)6 \Rightarrow l = 25 + 78 \Rightarrow l = 103$

b) $a = 5$, $d = 1.7$ find the 50th term

$l = a + (n-1)d \Rightarrow l = 5 + (50-1)1.7 \Rightarrow l = 5 + (49)1.7 \Rightarrow l = 5 + 83.3 \Rightarrow l = 88.3$

c) $a = 13$, $t_n = -52$, n = 14, $d = ?$

$l = a + (n-1)d \Rightarrow -52 = 13 + (14-1)d \Rightarrow -65 = 13d \Rightarrow d = -5$

d) $t_9 = 4$ $d = -2$, a = ?

$l = a + (n-1)d \Rightarrow 4 = a + (9-1)(-2) \Rightarrow 4 = a + (8)(-2) \Rightarrow 4 = a - 16 \Rightarrow a = 20$

4. Insert the specified number of arithmetic means between the given numbers.

a) One between 19 and 30.

$$\frac{a+l}{2} = \frac{19+30}{2} = \frac{49}{2} = 24.5$$

b) Five between -147 and 42

$a = -147, n = 7, l = 42$

$l = a + (n-1)d \Rightarrow 42 = -147 + (7-1)d \Rightarrow 189 = (6)d \Rightarrow d = {}^{189}\!/_{6} = {}^{63}\!/_{2}$

$-147, \underline{-147 + {}^{63}\!/_{2} = \frac{-294+63}{2} = \frac{-231}{2}}, \underline{\frac{-231}{2} + {}^{63}\!/_{2} = \frac{-168}{2}}, \underline{+{}^{63}\!/_{2} = \frac{-105}{2}},$

$\underline{\frac{-105}{2} + {}^{63}\!/_{2} = \frac{-42}{2}}, \underline{\frac{-42}{2} + {}^{63}\!/_{2} = \frac{21}{2}}, 42$

5. Find the sum of a series where $n = 25$, $a = 7$, $l = 23$

$$S_n = \frac{n}{2}[a+l] \Rightarrow S_n = \frac{25}{2}[7+23] \Rightarrow S_n = \frac{25}{2}[30] \Rightarrow S_n = 375$$

6. Find the sum of the series where $d = 1.8$, $a = 6$, $n = 100$

$$S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow S_n = \frac{100}{2}[2(6)+(100-1)(1.8)] \Rightarrow S_n = 50[12+(99)(1.8)] \Rightarrow$$

$$S_n = 50[12+178.2] \Rightarrow S_n = 50[190.2] \Rightarrow S_n = 9510$$

7. Summarize: 5 + 8 + 11 + 14 + … to 30 terms

$$\sum_{n=1}^{30} 5 + (n-1)3 = \sum_{n=1}^{30} 5 + 3n - 3 = \sum_{n=1}^{30} 2 + 3n$$

8. Expand and find the sum of $\sum_{n=1}^{12} 6 - 7n$

$(-1) + (-8) + (-15) + (-22) + ... + (-78)$

$$S_n = \frac{n}{2}[a+l] \Rightarrow S_n = \frac{12}{2}[(-1)+(-78)] \Rightarrow S_n = 6[-79] \Rightarrow S_n = -474$$

9. In a certain three digit number, the digits form an AP whose sum is 21. If the digits are reversed in order, the number is increased by 396. Find the number.

Equation One - sum of the digits

$(a)+(a+d)+(a+2d)=21 \Rightarrow 3a+3d=21$

Equation 2 - comparison of the digits in original and reverse order

$100(a+2d)+10(a+d)+1(a)=396+100(a)+10(a+d)+1(a+2d) \Rightarrow$

$100a+200d+10a+10d+a=396+100a+10a+10d+a+2d$

$111a+210d=396+111a+12d$

$198d=396$

$d=2$

$3a+3(2)=21 \Rightarrow 3a=15 \Rightarrow a=5$

Original number $=100(a)+10(a+d)+1(a+2d) \Rightarrow 100(5)+10(5+2)+1(5+2(2)) \Rightarrow$

$500+70+9=579$

Digits in Reverse Order = 975 Difference =396

10. Find the missing elements $n=11, l=-13, S_n=-33$

Find the value of "a"

$S_n=\frac{n}{2}[a+l] \Rightarrow -33=\frac{11}{2}[a+(-13)] \Rightarrow -66=11[a-13] \Rightarrow -6=a-13 \Rightarrow a=7$

Find the value of "d"

$l=a+(n-1)d \Rightarrow -13=7+(11-1)d \Rightarrow -20=10d \Rightarrow d=-2$