

Arithmetic Sequences and Series

1. A man takes a job at \$24,000 per year. He receives annual increases in pay of \$950. What is his salary during his seventh year on the job?

$$l = a + (n-1)d \Rightarrow l = 24000 + (7-1)(950) \Rightarrow l = 24000 + 6(950) \Rightarrow l = 29700$$

2. A Christmas Savings Club, which helps members to save money for Christmas shopping, requires each member to deposit \$25 the first week and to increase the deposit by \$8 weekly for nine weeks. How much is the final deposit?

$$l = a + (n-1)d \Rightarrow l = 25 + (9-1)(8) \Rightarrow l = 25 + 8(8) \Rightarrow l = 89$$

3. A college student typing a research paper finds that she can type 5 words per minute each 1/2 hour she types. If she starts at 37 words a minute at 7:30 p.m., how fast is she typing at 9:00 p.m.?

$$l = a + (n-1)d \Rightarrow l = 37 + (4-1)(5) \Rightarrow l = 37 + 3(5) \Rightarrow l = 52$$

4. A ball which rolls off a penthouse terrace falls 16 feet the first second, 48 feet the second, and 80 on the third. If it continues to fall in this manner, how far does it fall the seventh second.?

$$l = a + (n-1)d \Rightarrow l = 16 + (7-1)(32) \Rightarrow l = 16 + 6(32) \Rightarrow l = 208$$

5. The third term of an A.P. is 14 and the ninth term is -1. Find the first three terms.

Create a mini sequence with $a = 14, l = -1, n = 7$

$$l = a + (n-1)d \Rightarrow -1 = 14 + (7-1)(d) \Rightarrow -1 = 14 + 6d \Rightarrow d = -\frac{15}{6} = -\frac{5}{2}$$

$$l = a + (n-1)d \Rightarrow -1 = a + (9-1)(-\frac{5}{2}) \Rightarrow -1 = a + 8(-\frac{5}{2}) \Rightarrow -1 = a + -20 \Rightarrow a = 19$$

$$1\text{st term} = 19, 2\text{nd term} = 19 + (-\frac{5}{2}) = \frac{38}{2} - \frac{5}{2} = \frac{33}{2} = 16 \frac{1}{2}, 3\text{rd term} = 14$$

6. The seven weights in a set for an analytic balance are in an arithmetic progression. If the largest is 25 grams and the smallest is 1 gram, what are the other weights?

arithmetic means

$$l = a + (n-1)d \Rightarrow 1 = 25 + (7-1)d \Rightarrow 1 = 25 + 6d \Rightarrow -24 = 6d \Rightarrow d = -4$$

$$25, 21, 17, 13, 9, 5, 1$$

7. A young man's salary increased for 5 years in an arithmetic sequence. If his salary the first year was \$30,000 and his salary in the fifth year was \$45,000, what was his salary during each of the other years?

arithmetic means

$$l = a + (n-1)d \Rightarrow 45000 = 30000 + (5-1)d \Rightarrow 45000 = 30000 + (4)d \Rightarrow$$

$$15000 = 4d \Rightarrow d = 3750$$

$$30000, 33750, 37500, 41250, 45000$$

8. The arithmetic mean of two numbers is 9. If the sum of the squares of the numbers is 50 more than the square of 20, find the numbers.

Remember an arithmetic mean is the average of the two numbers

Two unknowns require two equations

$$a, l \Rightarrow 9 = \frac{a+l}{2} \Rightarrow 18 = a+l \Rightarrow a = 18-l$$

$$a^2 + l^2 = 50 + 20^2 \Rightarrow a^2 + l^2 = 450$$

Solve using substitution and factoring

$$(18-l)^2 + l^2 = 450 \Rightarrow 324 - 36l + l^2 + l^2 = 450 \Rightarrow 2l^2 - 36l - 126 = 0 \Rightarrow$$

$$2(l^2 - 18l - 63) = 0 \Rightarrow 2(l-21)(l+3) = 0 \therefore l = -3 \text{ or } 21$$

$$\therefore a = 18-l \Rightarrow a = 18-(-3) = 21 \text{ or } a = 18-(21) = -3$$

$$\frac{-3+21}{2} = 9 \text{ and } \frac{21+(-3)}{2} = 9$$

9. How much did an aeronautical engineer earn in ten years if her starting salary was \$40,000 and she received annual increases of \$1800.

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_n = \frac{10}{2}[2(40000) + (10-1)1800] \Rightarrow S_n = 5[80000 + 9(1800)] \Rightarrow$$

$$S_n = 5[80000 + 16200] \Rightarrow S_n = 5[96200] \Rightarrow S_n = 481000$$

10. In the front row of the physics lecture hall, there are 25 seats in the first row and 2 seats more in each following row. How many seats are there in the front 10 rows?

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_n = \frac{10}{2}[2(25) + (10-1)2] \Rightarrow S_n = 5[50 + 9(2)] \Rightarrow$$

$$S_n = 5[50 + 18] \Rightarrow S_n = 5[68] \Rightarrow S_n = 340$$

11. John repays the \$675 he owes his brother by making monthly payments. If his first payment is \$10 and every other payment is \$5 more than the preceding month, how long does it take him to pay off the debt?

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 675 = \frac{n}{2}[2(10) + (n-1)5] \Rightarrow 675 = \frac{n}{2}[20 + 5n - 5] \Rightarrow$$

$$1350 = n[15 + 5n] \Rightarrow 1350 = 15n + 5n^2 \Rightarrow 5n^2 + 15n - 1350 = 0 \Rightarrow 5(n^2 + 3n - 270) = 0 \Rightarrow$$

$$5(n+18)(n-15) = 0 \Rightarrow n = -18, n = 15$$

the answer is 15 months because "n" cannot be negative or a fraction

12. Find the missing terms:

a) $a = 3, l = 17, S_n = 100$

must determine n and d

$$S_n = \frac{n}{2}[a+l] \Rightarrow 100 = \frac{n}{2}[3+17] \Rightarrow 100 = \frac{n}{2}[20] \Rightarrow 100 = 10n \Rightarrow n = 10$$

$$l = a + (n-1)d \Rightarrow 17 = 3 + (10-1)d \Rightarrow 17 = 3 + 9d \Rightarrow 14 = 9d \Rightarrow d = 14/9$$

b) $n = 12, l = 1, S_n = -24$

must determine a and d

$$S_n = \frac{n}{2}[a+l] \Rightarrow -24 = \frac{12}{2}[a+1] \Rightarrow -24 = 6[a+1] \Rightarrow -24 = 6a+6 \Rightarrow -30 = 6a \Rightarrow a = -5$$

$$l = a + (n-1)d \Rightarrow 1 = -5 + (12-1)d \Rightarrow 1 = -5 + 11d \Rightarrow 6 = 11d \Rightarrow d = 6/11$$

c) $d = 5, n = 11, S_n = 275$

must determine a and l

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 275 = \frac{11}{2}[2a + (11-1)5] \Rightarrow 550 = 11[2a + 10(5)] \Rightarrow 550 = 11[2a + 50] \Rightarrow$$

$$550 = 22a + 550 \Rightarrow 0 = 22a \Rightarrow a = 0$$

$$l = a + (n-1)d \Rightarrow l = 0 + (11-1)5 \Rightarrow l = 19(5) \Rightarrow l = 50$$

13. Write using summation notation:

a) $2 + 5 + 8 + \dots$ to 12 terms terms $\sum_{n=1}^{12} a + (n-1)d \Rightarrow \sum_{n=1}^{12} 2 + (n-1)3 \Rightarrow \sum_{n=1}^{12} -1 + 3n$

b) $12 + 9 + 6 + 3 + 0 + (-3) + (-6) + (-9) + (-12)$

determine the number of terms

$$l = a + (n-1)d \Rightarrow -12 = 12 + (n-1)(-3) \Rightarrow -12 = 12 - 3n + 3 \Rightarrow -27 = -3n \Rightarrow n = 9$$

$$\sum_{n=1}^9 a + (n-1)d \Rightarrow \sum_{n=1}^9 12 + (n-1)(-3) \Rightarrow \sum_{n=1}^9 15 - 3n$$