Applications of Logs and Natural Logs:

1. If you start a biology experiment with 5,000,000 cells and 45% of the cells are dying every minute, how long will it take to have less than 1,000 cells?

$N = N_o k^t \Rightarrow 5{,}000{,}000 = 1000(.55)^t \Rightarrow .0002 = (.55)^t \Rightarrow \log .0002 = t \log .55 \Rightarrow t = 14.24$

2. A city in Midwestern United States had a population of 20,000 in 1900 and a population of 5,000 in 1995. The population decline followed an exponential model. What was the population in 1950? If we can assume that the model holds into the future, in what year will the city have less than 10 people.

First determine population growth constant:

$N = N_o k^t \Rightarrow 5{,}000 = 20{,}000k^{96} \Rightarrow 0.25 = k^{96} \Rightarrow \log .25 = 96 \log k \Rightarrow k = 0.9856$

Use the value of 'k" to solve remaining parts:

$N = N_o k^t \Rightarrow N = 20{,}000(.9856)^{51} \Rightarrow N = 9544$

$N = N_o k^t \Rightarrow 10 = 20{,}000(.9856)^t \Rightarrow 0.0005 = (.9856)^t \Rightarrow \log .0005 = t \log .9856 \Rightarrow 524.03$

3. If you start a biology experiment with 1,000,000 cells and one quarter of the cells are dying every 10 minutes, how long will it take to have less than 1,000 cells?

$N = N_o k^t \Rightarrow 1{,}000 = 1{,}000{,}000(.75)^t \Rightarrow .001 = .75^t \Rightarrow \log .001 = t \log .75 \Rightarrow$

$t = 24.01$ blocks on 10 minutes

Since a quarter of the pop dies every 10 min, 75% remain alive , therefore k = .75

4. If a Canadian city had a population of 120,000 in 1945 and a population of 265,000 in 1996, how large was the population in 1960 assuming an exponential population model?

$N = N_o k^t \Rightarrow 265{,}000 = 120{,}000k^{52} \Rightarrow 2.2083 = k^{52} \Rightarrow \log 2.2083 = 52 \log k \Rightarrow k = 1.015$

$N = 120{,}000(1.015)^{16} = 152.276$

5. If the population of a town in rural Saskatchewan had a population of 650 individuals in 1998 and the population is decreasing at a rate of 6% per year, determine:
    a. The population in 2005
    b. The projected population in 2020.

$N = N_o k^t \Rightarrow N = 650(.94)^8 \Rightarrow N = 396$

$N = 650(.94)^{23} = 156$

6. A biologist is researching a newly-discovered species of bacteria. At time t = 0 hours, he puts two hundred bacteria into what he has determined to be a favorable growth medium. Twelve hours later, he measures 2450 bacteria. Assuming exponential growth,
   a. Determine the growth constant "k" per hour for the bacteria? (Round k to two decimal places.)
   b. What would be the population size after 40 hours?

$N = N_O k^t \Rightarrow 2450 = 200k^{13} \Rightarrow 12.25 = k^{13} \Rightarrow \log 12.25 = 13 \log k \Rightarrow k = 1.2125$

$N = 200(1.2125)^{41} \Rightarrow N = 587{,}026$

7. Suppose that there were 90,000 bacteria present at the end of two days and 202,500 present at the end of four days.
   a. Find the number present at the beginning of the count.
   b. Find the number present at the end of 5 days.
   c. Find the approximate number of days at the end of which there are 560,000 bacteria.

$N = N_o k^t \Rightarrow 202{,}500 = 90{,}000k^3 \Rightarrow 2.25 = k^3 \Rightarrow \log 2.25 = 3\log k \Rightarrow 1.3103$

$a)\ 90{,}000 = N_o(1.3103)^3 \Rightarrow N_o = 40{,}006$

$b)\ N = 40{,}006(1.3103)^6 \Rightarrow N = 202464$

$c)\ 560{,}000 = 40{,}006(1.3103)^t \Rightarrow 13.9979 = 1.3103^t \Rightarrow \log 13.9979 = t \log 1.3103 \Rightarrow t = 9.7644$

8. Hospitals utilize the radioactive substance iodine-131 in the diagnosis of conditions of the thyroid gland. The half-life of iodine-131 is eight days. If a hospital acquires 4 g of iodine-131,
   a. How much of this sample will remain after 30 days ?
   b. How long will it be until only 0.02 g remains?

$a) N = N_o 2^{\frac{-x}{T}} \Rightarrow N = 4 \cdot 2^{\frac{-30}{8}} \Rightarrow N = 0.2973$

$b) 0.2 = 4 \cdot 2^{\frac{-x}{8}} \Rightarrow 0.05 = 2^{\frac{-x}{8}} \Rightarrow \log .05 = {}^{-x}\!/_{8} \log 2 \Rightarrow x = 34.5754$

9. The radioactive isotope sodium-24 is used as a tracer to measure the rate of flow in an artery or vein. The half-life of sodium-24 is 14.9 hours. Suppose that a hospital buys a 50-g sample of sodium-24 and will reorder when the sample is reduced to 4 g.
   a. How much of the sample will remain after 50 hours?
   b. How long before the hospital has to reorder sodium-24?
   c. How much of the sample will remain after 1 year?

$$a)\ N = N_o 2^{\frac{-x}{t}} \Rightarrow N = 50 \cdot 2^{\frac{-50}{14.9}} \Rightarrow N = 4.8842$$

$$b)\ 4 = 50 \cdot 2^{\frac{-x}{14.9}} \Rightarrow .08 = 2^{\frac{-x}{14.9}} \Rightarrow \log .08 = \frac{-x}{14.9} \log 2 \Rightarrow x = 54.29$$

$$c)\ N = 50 \cdot 2^{\frac{-8760}{14.9}} \Rightarrow N = 5.21 \times 10^{-176}$$

10. If a radioactive substance has a half-life of 300 years, how long will it take for the substance to decay to 10 per cent of its initial amount?

$$N = N_o 2^{\frac{-x}{T}} \Rightarrow .1 = 1 \cdot 2^{\frac{-x}{300}} \Rightarrow \log .1 = \frac{-x}{300} \log 2 \Rightarrow x = 996.578$$

11. In living matter the proportion of carbon which is radioactive does not vary with time (supposedly), but it decays from the time of death with a half-life of 5600 years. Date a piece of wood in which the radioactive carbon is 0.78 of the radioactive carbon of a similar piece of wood.

$$N = N_o 2^{\frac{-x}{T}} \Rightarrow .78 = 1 \cdot 2^{\frac{-x}{5600}} \Rightarrow \log .78 = \frac{-x}{5600} \log 2 \Rightarrow x = 2007.34$$

12. The half-life of a radioactive substance is one hour. What fraction of the substance is radioactive at the end of one second?, at the end of 3 seconds?

$$a)\ N = N_o 2^{\frac{-x}{T}} \Rightarrow N = 1 \cdot 2^{\frac{-1}{3600}} \Rightarrow N = 0.9998$$

$$b)\ N = 1 \cdot 2^{\frac{-3}{3600}} \Rightarrow N = 0.9994$$

13. Determine the worth of an investment after 10 years if the initial investment was \$800 at an interest rate of 7.5%

$$A = Pe^{rt} \Rightarrow A = 800 \cdot e^{.075 \cdot 10} \Rightarrow A = 1693.60$$

14. What amount would you have to invest to have a final amount of \$20,000 if it were invested at 8% for 16 years?

$$A = Pe^{rt} \Rightarrow 20000 = P \cdot e^{.08 \cdot 16} \Rightarrow 20000 = P \cdot 3.5966 \Rightarrow P = 5560.80$$

15. At what interest rate would you have to invest \$3,000 so that in 12 years you would have an amount of \$7,000?

$$A = Pe^{rt} \Rightarrow 7000 = 3000 \cdot e^{r \cdot 12} \Rightarrow 2.3333 = e^{12r} \Rightarrow \ln 2.3333 = 12r \ln e \Rightarrow$$

$$r = 0.0706 \Rightarrow r = 7.06\%$$

16. How many years would it take for $5,000 invested at 5.5% to result in a final amount of $9,000?

$A = Pe^{rt} \Rightarrow 9000 = 5000 \cdot e^{.055t} \Rightarrow 1.8 = e^{.055t} \Rightarrow \ln 1.8 = .055t \ln e \Rightarrow t = 10.687$

17. What would be the required interest rate for an investment to triple in 16 years?

$A = Pe^{rt} \Rightarrow 3 = 1 \cdot e^{16r} \Rightarrow \ln 3 = 16r \ln e \Rightarrow r = 0.06866 \Rightarrow r = 6.866\%$