A. Solve the following exponential equations:

1. $9^{5x+3} = 9^{-3x+7} \Rightarrow 5x+3 = -3x+7 \Rightarrow x = 4/8 = 1/2$
2. $64^{3x} = 16^{2x-9} \Rightarrow \left(2^6\right)^{3x} = \left(2^4\right)^{2x-9} \Rightarrow x = -36/10 = -18/5$
3. $\left(3^{2x}\right)^{x+1} = 27^{x+3} \Rightarrow \left(3^{2x}\right)^{x+1} = \left(3^3\right)^{x+3} \Rightarrow x = \dfrac{1 \pm \sqrt{73}}{4}$
4. $\left(49^x\right)^{x+1} = \left(7^{x-3}\right)^2 \Rightarrow \left(7^{2x}\right)^{x+1} = \left(7^{x-3}\right)^2 \Rightarrow x = \varnothing$
5. $25^{3x-7} \cdot 625^{6x+3} = 125^{2x} \cdot 5^{5x-7} \Rightarrow \left(5^2\right)^{3x-7} \cdot \left(5^4\right)^{6x+3} = \left(3^5\right)^{2x} \cdot 5^{5x-7} \Rightarrow x = -5/15 = -1/3$

B. Solve the following log equations:

1. $\log_2 x = 7 \Rightarrow x = 2^7 = 128$
2. $\log_x 5 = 3 \Rightarrow 5 = x^3 \Rightarrow x = \sqrt[3]{5}$
3. $\log_3 7 = x \Rightarrow 7 = 3^x \Rightarrow \log 7 = x \log 3 \Rightarrow x = 1.7712$
4. $\log_5 2 - \log_5 8 = \log_5 x \Rightarrow \log_5 2/8 = \log_5 x \Rightarrow x = 2/8 = 1/4$
5. $\log 7 + \log x = \log 49 \Rightarrow \log 7x = \log 49 \Rightarrow 7x = 49 \Rightarrow x = 7$
6. $8 \log x - 3 \log x = \log 32 \Rightarrow \log \dfrac{x^8}{x^3} = \log 32 \Rightarrow x^5 = 32 \Rightarrow x = 2$
7. $\log\left(x^2 + 21x\right) = 2 \Rightarrow x^2 + 21x = 10^2 \Rightarrow x^2 + 21x - 100 = 0 \Rightarrow x = -25,\ 4$
8. $2 = \log_3(9x+10) - \log_3(5x) \Rightarrow 2 = \log_3 \dfrac{9x+10}{5x} \Rightarrow 3^2 = \dfrac{9x+10}{5x} \Rightarrow x = 10/36 = 5/18$
9. $\log_3\left(x^3 - 1\right) - \log_3\left(x^2 + x + 1\right) = \log_3 12 \Rightarrow \log_3 \dfrac{\left(x^3 - 1\right)}{\left(x^2 + x + 1\right)} = \log_3 12 \Rightarrow$
$$\frac{(x-1)\left(x^2 + x + 1\right)}{\left(x^2 + x + 1\right)} = 12 \Rightarrow x = 13$$
10. $\log_3(x-3) + \log_3(x+2) = 7 \Rightarrow \log_3(x-3)(x+2) = 7 \Rightarrow (x-3)(x+2) = 3^7 \Rightarrow$
$$x^2 - x - 2187 = 0 \Rightarrow x = \frac{1 + \sqrt{8773}}{2}$$
11. $\log\left(3x^2 + 4\right) - \log(2x-2) = \log x + \log 4 \Rightarrow \log \dfrac{\left(3x^2 + 4\right)}{(2x-2)} = \log \dfrac{x}{4} \Rightarrow \dfrac{\left(3x^2 + 4\right)}{(2x-2)} = \dfrac{x}{4} \Rightarrow x = \varnothing$
12. $\log_8 11 = \log_3 x \Rightarrow \dfrac{\log 11}{\log 8} = \dfrac{\log x}{\log 3} \Rightarrow \log x = 0.5501 \Rightarrow x = 3.5489$
13. $\ln(x+1) - \ln x = \ln 2 \Rightarrow \ln \dfrac{(x+1)}{x} = \ln 2 \Rightarrow \dfrac{(x+1)}{x} = 2 \Rightarrow x = 1$

14. $\ln x + 3\sqrt{\ln x} = 10 \Rightarrow \ln x + \frac{3}{2}\ln x = 10 \Rightarrow \ln\left(x \cdot x^{\frac{3}{2}}\right) = 10 \Rightarrow x^{\frac{5}{2}} = e^{10}$

$\Rightarrow \left(x^{\frac{5}{2}}\right)^{\frac{2}{5}} = \left(e^{10}\right)^{\frac{2}{5}} \Rightarrow x = e^4$

15. $\ln(8x^3 - 216) - \ln(4x^2 + 12x + 36) = 2 \Rightarrow \ln\frac{(8x^3 - 216)}{(4x^2 + 12x + 36)} = 2 \Rightarrow$

$\frac{(2x-6)(4x^2 + 12x + 36)}{(4x^2 + 12x + 36)} = e^2 = 2.7183^2 \Rightarrow 2x - 6 = 7.3890 \Rightarrow x = 6.6945$

16. $6\ln x - 4\ln x = 4 \Rightarrow \ln\frac{x^6}{x^4} = 4 \Rightarrow x^2 = e^4 = 2.7183^4 = 54.5981 \Rightarrow x = \sqrt{54.5981}$

C. Solve the following exponential equations (use logs on questions 1- 4 and natural logs on questions 5 – 8)

1. $5^{4x+1} = 13 \Rightarrow (4x+1)\log 5 = \log 13 \Rightarrow x = 0.1484$
2. $7^{2x} = 11 \cdot 3^{x-1} \Rightarrow 2x\log 7 = \log 11 + (x-1)\log 3 \Rightarrow x = 0.4651$
3. $121^{4x+1} = 35(8^{3x}) \Rightarrow (4x+1)\log 121 = \log 35 + 3x\log 8 \Rightarrow x = -0.0958$
4. $26^{(x^2+2x)} = 703 \Rightarrow (x^2 + 2x)\log 26 = \log 703 \Rightarrow x^2 + 2x - 2.0120 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{12.048}}{2}$
5. $3^{3x} = 11^{x+1} \Rightarrow 3x(\ln 3) = (x+1)\ln 11 \Rightarrow x = 2.67$
6. $17^{2x-5} = 31 \cdot 2^{-3x+6} \Rightarrow (2x-5)\ln 17 = \ln 31 + (-3x+6)\ln 2 \Rightarrow x = 2.809$
7. $8^{(2x^2+3)} = 13^{(5x-1)} \Rightarrow (2x^2+3)\ln 8 = (5x-1)\ln 13 \Rightarrow$

   $4.1588x^2 - 6.5864x + 2.5649 = 0 \Rightarrow x = \frac{6.5864 \pm \sqrt{.7130}}{8.3176}$
8. $5^{\frac{1}{2}x} \cdot 3^{\frac{2}{3}} = \sqrt[5]{7^{2x}} \Rightarrow 5^{\frac{1}{2}x} \cdot 3^{\frac{2}{3}} = 7^{\frac{2}{5}x} \Rightarrow \frac{1}{2}x\ln 5 + \frac{2}{3}\ln 3 = \frac{2}{5}x\ln 7 \Rightarrow x = 27.7424$

D. 1. $N = N_o k^t \Rightarrow 4100 = 600(3)^t \Rightarrow 6.8333 = 3^t \Rightarrow \log 6.8333 = t\log 3 \Rightarrow t = 1.7492$
Number of days $5(1.7492) = 8.7463$

2. $N = N_o(2)^{-\frac{x}{T}} \Rightarrow 6 = 4781(2)^{-\frac{500}{T}} \Rightarrow 0.0012 = (2)^{-\frac{500}{T}} \Rightarrow$

   $\log .0012 = -\frac{500}{T}\log 2 \Rightarrow T = 51.5317$

3. $A = Pe^{rt} \Rightarrow 3600 = 1200e^{r\cdot 17} \Rightarrow \log 3 = 17r\log 2.7183 \Rightarrow x = 0.0646 = 6.46\%$

$$\frac{125 \cdot 0.0345 \cdot \sqrt[4]{513}}{1698 \cdot \sqrt{8912} \cdot (0.014)^{2/3}}$$

$$\log 125 = \qquad 2.0969$$

$$\log 0.0345 = \qquad -1.4621$$

$$\frac{1}{4}\log 513 = \frac{1}{4}(2.7101) = 0.6775$$

E. $\qquad 1.3123 \qquad$ Antilog $(1.3123 - 4.0381) = 0.0018$

$$\log 1698 = \qquad 3.2299$$

$$\frac{1}{2}\log 8912 = \frac{1}{2}(3.9499) = \qquad 1.9749$$

$$\frac{2}{3}\log 0.014 = \frac{2}{3}(-1.8538) = -1.2358$$

$$4.0381$$

F. a) $y = -3^x \Rightarrow$ graph 4 b) $y = \ln x \Rightarrow$ graph 2

c) $y = e^x \Rightarrow$ graph 1 d) $y = \log x \Rightarrow$ graph 3

G. Natural log (ln) is the log to the base “e” and can be defined as $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ and can be calculated by finding the sum $e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$