

Exponential Worksheet

Simplified exponential answer comes first and simplified radical comes after the arrow

A. Express each answer in simplest radical form:

$$1. \left(\sqrt[3]{\frac{16x^5}{54y^2}} \right)^2 = \left(\left(\frac{2^4 x^5}{2 \cdot 3^3 y^2} \right)^{\frac{1}{3}} \right)^2 = \frac{2^{\frac{8}{3}} x^{\frac{10}{3}}}{2^{\frac{2}{3}} \cdot 3^{\frac{6}{3}} y^{\frac{4}{3}}} = \frac{2^{\frac{6}{3}} x^{\frac{10}{3}}}{3^{\frac{6}{3}} y^{\frac{4}{3}}} \Rightarrow \frac{\sqrt[3]{2^6 x^{10}}}{\sqrt[3]{3^6 y^4}} = \frac{\sqrt[3]{2^6 x^{10}}}{\sqrt[3]{3^6 y^4}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \frac{2^2 x^3 \sqrt[3]{xy^2}}{3^2 y^2}$$

$$2. \left(\sqrt[4]{\frac{27}{4x^2}} \right)^5 = \left(\left(\frac{3^3}{2^2 x^2} \right)^{\frac{1}{4}} \right)^5 = \frac{3^{\frac{15}{4}}}{2^{\frac{10}{4}} x^{\frac{10}{4}}} \Rightarrow \frac{\sqrt[4]{3^{15}}}{\sqrt[4]{2^{10} x^{10}}} = \frac{\sqrt[4]{3^{15}}}{\sqrt[4]{2^{10} x^{10}}} \cdot \frac{\sqrt[4]{2^2 x^2}}{\sqrt[4]{2^2 x^2}} = \frac{3^3 \sqrt[4]{2^2 \cdot 3^3 x^2}}{2^3 x^3}$$

$$3. \left(\sqrt[5]{\frac{16x^3}{27y^4}} \right)^2 = \left(\left(\frac{2^4 x^3}{3^3 y^4} \right)^{\frac{1}{5}} \right)^2 = \frac{2^{\frac{8}{5}} x^{\frac{6}{5}}}{3^{\frac{6}{5}} y^{\frac{8}{5}}} \Rightarrow \frac{\sqrt[5]{2^8 x^6}}{\sqrt[5]{3^6 y^8}} = \frac{\sqrt[5]{2^8 x^6}}{\sqrt[5]{3^6 y^8}} \cdot \frac{\sqrt[5]{3^4 y^2}}{\sqrt[5]{3^4 y^2}} = \frac{2x^5 \sqrt[5]{2^3 \cdot 3^4 xy^2}}{3^2 y^2}$$

$$4. \sqrt{\sqrt[3]{9a^2x^7}} = \left((3^2 a^2 x^7)^{\frac{1}{3}} \right)^{\frac{1}{2}} = 3^{\frac{2}{6}} a^{\frac{2}{6}} x^{\frac{7}{6}} \Rightarrow \sqrt[6]{3^2 a^2 x^7} = x \sqrt[6]{3^2 a^2 x}$$

$$5. \sqrt[4]{2\sqrt[3]{16x^3y}} = \left(2(2^4 x^3 y)^{\frac{1}{3}} \right)^{\frac{1}{4}} = \left(2 \cdot 2^{\frac{4}{3}} x^{\frac{3}{3}} y^{\frac{1}{3}} \right)^{\frac{1}{4}} = 2^{\frac{1}{4}} \cdot 2^{\frac{4}{3} \cdot \frac{1}{4}} x^{\frac{3}{3} \cdot \frac{1}{4}} y^{\frac{1}{3} \cdot \frac{1}{4}}$$

$$6. \sqrt[3]{4\sqrt{27x^6y^9}} = \left((3^3 x^6 y^9)^{\frac{1}{4}} \right)^{\frac{1}{3}} = 3^{\frac{3}{12}} x^{\frac{6}{12}} y^{\frac{9}{12}} \Rightarrow \sqrt[12]{3^3 x^6 y^9}$$

$$7. \sqrt[3]{2x^5y\sqrt{8x^3y^5}} = \left(2x^5y(2^3 x^3 y^5)^{\frac{1}{2}} \right)^{\frac{1}{3}} = \left(2x^5y \cdot 2^{\frac{3}{2}} x^{\frac{3}{2}} y^{\frac{5}{2}} \right)^{\frac{1}{3}} = 2^{\frac{1}{3}} x^{\frac{5}{3}} y^{\frac{1}{3}} \cdot 2^{\frac{3}{6}} x^{\frac{3}{6}} y^{\frac{5}{6}} = 2^{\frac{2}{6}} x^{\frac{10}{6}} y^{\frac{2}{6}} \cdot 2^{\frac{3}{6}} x^{\frac{3}{6}} y^{\frac{5}{6}} = 2^{\frac{5}{6}} x^{\frac{13}{6}} y^{\frac{7}{6}} \Rightarrow \sqrt[6]{2^5 x^{13} y^7} = x^2 y \sqrt[6]{2^5 xy}$$

$$8. \sqrt[4]{27x^2y^3\sqrt[3]{81x^6y^4}} = \left(3^3 x^2 y^3 (3^4 x^6 y^4)^{\frac{1}{3}} \right)^{\frac{1}{4}} = \left(3^3 x^2 y^3 \cdot 3^{\frac{4}{3}} x^{\frac{6}{3}} y^{\frac{4}{3}} \right)^{\frac{1}{4}} = 3^{\frac{3}{4}} x^{\frac{2}{4}} y^{\frac{3}{4}} \cdot 3^{\frac{4}{12}} x^{\frac{6}{12}} y^{\frac{4}{12}} = 3^{\frac{9}{12}} x^{\frac{6}{12}} y^{\frac{9}{12}} \cdot 3^{\frac{4}{12}} x^{\frac{6}{12}} y^{\frac{4}{12}} = 3^{\frac{13}{12}} x^{\frac{12}{12}} y^{\frac{13}{12}} \Rightarrow \sqrt[12]{3^{13} x^{12} y^{13}} = 3xy \sqrt[12]{3y}$$

$$9. \frac{\sqrt[3]{4a^2x^4}\sqrt[4]{8a^3x^2}}{\sqrt{2ax}\sqrt[6]{8a^4x^2}} = \frac{(2^2a^2x)^{\frac{1}{3}}(2^3a^3x^2)^{\frac{1}{4}}}{(2ax)^{\frac{1}{2}}(2^3a^4x^2)^{\frac{1}{6}}} = \frac{2^{\frac{2}{3}}a^{\frac{2}{3}}x^{\frac{1}{3}} \cdot 2^{\frac{3}{4}}a^{\frac{3}{4}}x^{\frac{2}{4}}}{2^{\frac{1}{2}}a^{\frac{1}{2}}x^{\frac{1}{2}} \cdot 2^{\frac{3}{6}}a^{\frac{4}{6}}x^{\frac{2}{6}}} = \frac{2^{\frac{8}{12}}a^{\frac{8}{12}}x^{\frac{4}{12}} \cdot 2^{\frac{9}{12}}a^{\frac{9}{12}}x^{\frac{6}{12}}}{2^{\frac{6}{12}}a^{\frac{6}{12}}x^{\frac{12}{12}} \cdot 2^{\frac{8}{12}}a^{\frac{8}{12}}x^{\frac{4}{12}}} =$$

$$\frac{2^{\frac{17}{12}}a^{\frac{17}{12}}x^{\frac{10}{12}}}{2^{\frac{12}{12}}a^{\frac{12}{12}}x^{\frac{10}{12}}} = 2^{\frac{5}{12}}a^{\frac{3}{12}} \Rightarrow \sqrt[12]{2^5a^3}$$

$$10. \sqrt{\sqrt[3]{x^2y^4}\sqrt{x^3y^2}} = \left((x^2y)^{\frac{1}{3}}(x^3y^2)^{\frac{1}{4}} \right)^{\frac{1}{2}} = \left(x^{\frac{2}{3}}y^{\frac{1}{3}} \cdot x^{\frac{3}{4}}y^{\frac{2}{4}} \right)^{\frac{1}{2}} = x^{\frac{2}{6}}y^{\frac{1}{6}} \cdot x^{\frac{3}{8}}y^{\frac{2}{8}} =$$

$$x^{\frac{8}{24}}y^{\frac{4}{24}} \cdot x^{\frac{9}{24}}y^{\frac{6}{24}} = x^{\frac{17}{24}}y^{\frac{10}{24}} \Rightarrow \sqrt[24]{x^{17}y^{10}}$$

$$11. \sqrt[5]{\frac{25x^4y^2}{\sqrt[7]{625x^3y^4}}} = \left(\frac{(5^2x^4y^2)^{\frac{1}{5}}}{(5^4x^3y^4)^{\frac{1}{7}}} \right)^{\frac{1}{5}} = \left(\frac{5^{\frac{2}{4}}x^{\frac{4}{3}}y^{\frac{2}{4}}}{5^{\frac{4}{7}}x^{\frac{3}{7}}y^{\frac{4}{7}}} \right)^{\frac{1}{5}} = \frac{5^{\frac{2}{5}}x^{\frac{4}{5}}y^{\frac{2}{5}}}{5^{\frac{4}{5}}x^{\frac{3}{5}}y^{\frac{4}{5}}} = \frac{5^{\frac{14}{35}}x^{\frac{28}{35}}y^{\frac{14}{35}}}{5^{\frac{4}{35}}x^{\frac{3}{35}}y^{\frac{4}{35}}} =$$

$$5^{\frac{10}{35}}x^{\frac{25}{35}}y^{\frac{10}{35}} \Rightarrow \sqrt[35]{5^{10}x^{25}y^{10}}$$

$$12. \sqrt[3]{\frac{4x^4\sqrt[4]{y^3}}{6x^2\sqrt{y}}} = \left(\frac{2^2x(y^3)^{\frac{1}{4}}}{2 \cdot 3x^2(y)^{\frac{1}{2}}} \right)^{\frac{1}{3}} = \left(\frac{2^2xy^{\frac{3}{4}}}{2 \cdot 3x^2y^{\frac{1}{2}}} \right)^{\frac{1}{3}} = \frac{2^{\frac{2}{3}}x^{\frac{1}{3}}y^{\frac{3}{12}}}{2^{\frac{1}{3}} \cdot 3^{\frac{1}{3}}x^{\frac{2}{3}}y^{\frac{1}{6}}} = \frac{2^{\frac{8}{12}}x^{\frac{4}{12}}y^{\frac{3}{12}}}{2^{\frac{4}{12}} \cdot 3^{\frac{4}{12}}x^{\frac{8}{12}}y^{\frac{2}{12}}} = \frac{2^{\frac{4}{12}}y^{\frac{1}{12}}}{3^{\frac{4}{12}}x^{\frac{4}{12}}} \Rightarrow$$

$$\frac{\sqrt[12]{2^4y}}{\sqrt[12]{3^4x^4}} = \frac{\sqrt[12]{2^4y}}{\sqrt[12]{3^4x^4}} \cdot \frac{\sqrt[12]{3^8x^8}}{\sqrt[12]{3^8x^8}} = \frac{\sqrt[12]{2^43^8x^8y}}{3x}$$

$$13. \sqrt[5]{3\sqrt[3]{27a^3x}} = \left(3(3^3a^3x)^{\frac{1}{3}} \right)^{\frac{1}{5}} = \left(3 \cdot 3^{\frac{3}{3}}a^{\frac{3}{3}}x^{\frac{1}{3}} \right)^{\frac{1}{5}} = 3^{\frac{1}{5}} \cdot 3^{\frac{3}{10}}a^{\frac{3}{10}}x^{\frac{1}{10}} = 3^{\frac{2}{10}} \cdot 3^{\frac{3}{10}}a^{\frac{3}{10}}x^{\frac{1}{10}} =$$

$$3^{\frac{5}{10}}a^{\frac{3}{10}}x^{\frac{1}{10}} \Rightarrow \sqrt[10]{3^5a^3x}$$

$$14. \sqrt[4]{9\sqrt[3]{9x^2y^4}} = \left(3^2(3^2x^2y^4)^{\frac{1}{3}} \right)^{\frac{1}{4}} = \left(3^2 \cdot 3^{\frac{2}{3}}x^{\frac{2}{3}}y^{\frac{4}{3}} \right)^{\frac{1}{4}} = 3^{\frac{2}{4}} \cdot 3^{\frac{2}{12}}x^{\frac{2}{12}}y^{\frac{4}{12}} = 3^{\frac{6}{12}} \cdot 3^{\frac{2}{12}}x^{\frac{2}{12}}y^{\frac{4}{12}} =$$

$$3^{\frac{8}{12}}x^{\frac{2}{12}}y^{\frac{4}{12}} \Rightarrow \sqrt[12]{3^8x^2y^4}$$

$$15. \sqrt[5]{\frac{\sqrt[3]{a^2b}}{\sqrt{ab}}} = \left(\frac{(a^2b)^{\frac{1}{3}}}{(ab)^{\frac{1}{2}}} \right)^{\frac{1}{5}} = \left(\frac{a^{\frac{2}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{2}}b^{\frac{1}{2}}} \right)^{\frac{1}{5}} = \frac{a^{\frac{2}{15}}b^{\frac{1}{15}}}{a^{\frac{1}{10}}b^{\frac{1}{10}}} = \frac{a^{\frac{4}{30}}b^{\frac{2}{30}}}{a^{\frac{3}{30}}b^{\frac{3}{30}}} = \frac{a^{\frac{1}{30}}}{b^{\frac{1}{30}}} \Rightarrow \frac{\sqrt[30]{a}}{\sqrt[30]{b}} = \frac{\sqrt[30]{a}}{\sqrt[30]{b}} \cdot \frac{\sqrt[30]{b^{29}}}{\sqrt[30]{b^{29}}} = \frac{\sqrt[30]{ab^{29}}}{b}$$

$$16. \frac{\sqrt{2a^3}\sqrt[4]{4a^2b^2}}{\sqrt[6]{16a^4b^3}} = \frac{(2a)^{\frac{1}{2}}(2^2a^2b^2)^{\frac{1}{3}}}{(2^4a^4b^3)^{\frac{1}{6}}} = \frac{2^{\frac{1}{2}}a^{\frac{1}{2}} \cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}b^{\frac{2}{3}}}{2^{\frac{4}{6}}a^{\frac{4}{6}}b^{\frac{3}{6}}} = \frac{2^{\frac{3}{6}}a^{\frac{3}{6}} \cdot 2^{\frac{4}{6}}a^{\frac{4}{6}}b^{\frac{4}{6}}}{2^{\frac{4}{6}}a^{\frac{4}{6}}b^{\frac{3}{6}}} = \frac{2^{\frac{7}{6}}a^{\frac{4}{6}}b^{\frac{4}{6}}}{2^{\frac{4}{6}}a^{\frac{4}{6}}b^{\frac{3}{6}}} = 2^{\frac{3}{6}}b^{\frac{1}{6}} \Rightarrow \sqrt[6]{2^3b}$$

17.

$$\frac{\sqrt{15x^4y^2}\sqrt[3]{75x^2y^3}}{\sqrt[4]{45x^6y^3}} = \frac{(3 \cdot 5x^4y^2)^{\frac{1}{2}}(3 \cdot 5^2x^2y^3)^{\frac{1}{3}}}{(3^2 \cdot 5x^6y^3)^{\frac{1}{4}}} = \frac{3^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}x^2y^2 \cdot 3^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}x^{\frac{2}{3}}y^3}{3^{\frac{2}{4}} \cdot 5^{\frac{1}{4}}x^{\frac{6}{4}}y^{\frac{3}{4}}} = \frac{3^{\frac{6}{12}} \cdot 5^{\frac{8}{12}}x^{\frac{24}{12}}y^{\frac{24}{12}} \cdot 3^{\frac{4}{12}} \cdot 5^{\frac{8}{12}}x^{\frac{8}{12}}y^{\frac{12}{12}}}{3^{\frac{6}{12}} \cdot 5^{\frac{3}{12}}x^{\frac{18}{12}}y^{\frac{9}{12}}} = \frac{3^{\frac{10}{12}} \cdot 5^{\frac{14}{12}}x^{\frac{32}{12}}y^{\frac{24}{12}}}{3^{\frac{6}{12}} \cdot 5^{\frac{3}{12}}x^{\frac{18}{12}}y^{\frac{9}{12}}} = 3^{\frac{4}{12}} \cdot 5^{\frac{11}{12}}x^{\frac{14}{12}}y^{\frac{15}{12}} \Rightarrow \sqrt[12]{3^45^{11}x^{14}y^{15}} = xy^{12}\sqrt[12]{3^45^{11}x^2y^3}$$

$$18. \frac{x^{\frac{3}{2}}y^{\frac{5}{4}}z^{\frac{-3}{5}}}{x^{\frac{-7}{4}}y^{\frac{1}{5}}z^{\frac{-7}{2}}} = \frac{x^{\frac{3}{2}}y^{\frac{5}{4}}x^{\frac{7}{4}}z^{\frac{7}{2}}}{z^{\frac{3}{5}}y^{\frac{1}{5}}} = \frac{x^{\frac{30}{20}}y^{\frac{25}{20}}x^{\frac{35}{20}}z^{\frac{70}{20}}}{z^{\frac{12}{20}}y^{\frac{4}{20}}} = x^{\frac{65}{20}}y^{\frac{21}{20}}z^{\frac{58}{20}} \Rightarrow \sqrt[20]{x^{65}y^{21}z^{58}} = x^3y^2z^2\sqrt[20]{x^5yz^{18}}$$

$$19. \sqrt[3]{\sqrt[4]{\sqrt{x^{30}y^{70}z^{36}}}} = \left(\left((x^{30}y^{70}z^{36})^{\frac{1}{2}} \right)^{\frac{1}{4}} \right)^{\frac{1}{3}} = x^{\frac{30}{24}}y^{\frac{70}{24}}z^{\frac{36}{24}} \Rightarrow \sqrt[24]{x^{30}y^{70}z^{36}} = xy^2z^2\sqrt[24]{x^6y^{22}z^{12}}$$

$$20. \sqrt[5]{x^9\sqrt[3]{x^3y^7}\sqrt{x^5y^3}} = \left(x^9 \left(x^3y^7(x^5y^3)^{\frac{1}{2}} \right)^{\frac{1}{3}} \right)^{\frac{1}{5}} = \left(x^9 \left(x^3y^7x^{\frac{5}{2}}y^{\frac{3}{2}} \right)^{\frac{1}{3}} \right)^{\frac{1}{5}} = \left(x^9x^{\frac{3}{3}}y^{\frac{7}{3}}x^{\frac{5}{6}}y^{\frac{3}{6}} \right)^{\frac{1}{5}} = x^{\frac{9}{5}}x^{\frac{3}{5}}y^{\frac{7}{5}}x^{\frac{5}{30}}y^{\frac{3}{30}} = x^{\frac{54}{30}}x^{\frac{6}{30}}y^{\frac{14}{30}}x^{\frac{5}{30}}y^{\frac{3}{30}} = x^{\frac{65}{30}}y^{\frac{17}{30}} \Rightarrow \sqrt[30]{x^{65}y^{17}} = x^2\sqrt[30]{x^5y^{17}}$$