Simplifying Rational Expressions

Goal: to eliminate the same factors found in both the numerator and denominator. (In simple terms- reducing a fraction to its lowest terms)

Example #1: $\frac{15\,x^3y^2}{25\,x^2y^4}$

Steps: 1. Break all numbers into prime factors. $15 = 3\cdot 5 \quad and \quad 25 = 5\cdot 5 = 5^2$

Remember: prime factors are numbers such as 2, 3, 5, 7, 11, 13, 17, 19, ….
Variables and expressions (completely factored and written with enclosed parentheses) can also be considered as prime factors.

2. Re-write expression as product of prime factors: $\frac{15x^3y^2}{25x^2y^4} = \frac{3\cdot 5x^3y^2}{5^2x^2y^4}$

3. Use the Laws of Exponents to reduce the fractions. The most common rule will be to subtract exponents. You must remember to leave your answer with positive exponents.

$$\frac{15x^3y^2}{25x^2y^4} = \frac{3\cdot 5x^3y^2}{5^2x^2y^4} = \frac{3x}{5y^2}$$

Example #2: $\frac{36(3x-1)(2x+5)^3}{42(3x-1)^2(2x+5)}$

Steps: 1. Break all numbers into prime factors. $36 = 2\cdot 2\cdot 3\cdot 3 = 2^2\cdot 3^2 \quad and \quad 42 = 2\cdot 3\cdot 6$

2. Re-write expression as product of prime factors: $\frac{36(3x-1)(2x+5)^3}{42(3x-1)^2(2x+5)} = \frac{2^2\cdot 3^2\cdot (3x-1)(2x+5)^3}{2\cdot 3\cdot 6\cdot (3x-1)^2(2x+5)}$

3. Use the Laws of Exponents to reduce the fractions.

$$\frac{36(3x-1)(2x+5)^3}{42(3x-1)^2(2x+5)} = \frac{2^2\cdot 3^2\cdot (3x-1)(2x+5)^3}{2\cdot 3\cdot 7\cdot (3x-1)^2(2x+5)} = \frac{2\cdot 3\cdot (2x+5)^3}{7\cdot (3x-1)}$$

Example #3: $\frac{3^2(x-1)(2x+3)^3}{3^3(x-1)^2(2x+3)^3}$

Steps: 1. Break all numbers into prime factors. - all elements in the question are factored
2. Re-write expression as product of prime factors - is in that format
3. Use the Laws of Exponents to reduce the fractions.

$$\frac{3^2(x-1)(2x+3)^3}{3^3(x-1)^2(2x+3)^3} = \frac{1}{3(x-1)}$$

Note: if factors cancel in such a way as there appears to be no factor remaining in the numerator, you must still write your answer as a fraction and you must write the numerator with the factor "1".

Example #4: $\dfrac{3^4(x-1)^4(2x+3)^3}{3^3(x-1)^2(2x+3)^3}$

Steps: 1. Break all numbers into prime factors. - all elements in the question are factored
2. Re-write expression as product of prime factors - is in that format
3. Use the Laws of Exponents to reduce the fractions.

$$\frac{3^4(x-1)^4(2x+3)^3}{3^3(x-1)^2(2x+3)^3} = 3(x-1)^2$$

Note: if factors cancel in such a way as there appears to be no factor remaining in the denominator, you do not need to write your answer as a fraction. It is understood that the denominator has a value of "1" and that "anything" over "1" can be written as just "anything" and the "1" can be left out. $\left(\dfrac{5}{1} = 5,\ \dfrac{x}{1} = x,\ \dfrac{(x+1)}{1} = (x+1)\right)$

Example #5: $\dfrac{6x^4+10x^3+4x^2}{4x^3-4x}$

Steps: 1. Factor the numerator and denominator and write as a product of primes. If common factor are removed, remember to convert each one to prime factors.

$$\frac{6x^4+10x^3+4x^2}{4x^3-4x} = \frac{2x^2(3x+2)(x+1)}{2^2x(x+1)(x-1)}$$

2. Use the Laws of Exponents to reduce the fractions

$$\frac{6x^4+10x^3+4x^2}{4x^3-4x} = \frac{2x^2(3x+2)(x+1)}{2^2x(x+1)(x-1)} = \frac{x(3x+2)}{2(x-1)}$$