CIRCULAR PERMUTATIONS

Types of circular permutations:

a) stationary - table, people in a ring, etc. b) movable - key ring, necklace, charm bracelet

1. In how many ways can:

a) four people be seated at a table? $(n-1)! \Rightarrow (4-1)! \Rightarrow 3 \cdot 2 \cdot 1$

b) six people be seated at a table? $(n-1)! \Rightarrow (6-1)! \Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

c) twelve people be seated at a table? $(n-1)! \Rightarrow (12-1)! \Rightarrow 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

d) twenty-four students be arranged in a circle? $(n-1)! \Rightarrow (24-1)! \Rightarrow 23 \cdot 22 \cdot 21 \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

2. In how many ways can:

a) four people be seated at a table if:

i) two must sit together? $(n-1)! \cdot 2! \Rightarrow (3-1)!2! \Rightarrow 2 \cdot 1 \cdot 2 \cdot 1$

ii) three must sit together? $(n-1)! \cdot 3! \Rightarrow (2-1)!3! \Rightarrow 1 \cdot 3 \cdot 2 \cdot 1$

b) eight people be seated at a table if:

i) two must be seated together? $(n-1)! \cdot 2! \Rightarrow (7-1)!2! \Rightarrow 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1$

ii) three must be seated together? $(n-1)! \cdot 3! \Rightarrow (6-1)!3! \Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$

iii) five must be seated together? $(n-1)! \cdot 5! \Rightarrow (4-1)!5! \Rightarrow 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

3. In how many ways can:

a) four boys and four girls be arranged in a circle so that they alternate?

$(n-1)!n! \Rightarrow (4-1)!4! \Rightarrow 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

b) eight boys and eight girls be arranged in a circle so that they alternate?

$(n-1)!n! \Rightarrow (8-1)!8! \Rightarrow 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

c) ten boys be arranged in an outside ring and ten girls be arranged in an inside ring?

$(n-1)!n! \Rightarrow (10-1)!10! \Rightarrow 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

4. Ten people (5 men and 5 women) are attending a dinner party. In how many ways can they be arranged if:

a) they are to be arranged in a line for a picture? $10! \Rightarrow 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

b) they are arranged in a line for a picture but the man and women must alternate?

mfmfmfmfmf or *fmfmfmfmfm* $\Rightarrow 5 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 + 5 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1$

c) they are arranged in two lines, men in back and women in front, for a picture?

$m \Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

and $\Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$f \Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

d) they are to be arranged in one line but as couples?

couples then within couples $\Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1$

e) they are to be arranged around the dinner table? $(n-1)! \Rightarrow (10-1)! \Rightarrow 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

f) they are to be arranged with men and women alternating around the table?

$(n-1)!n! \Rightarrow (5-1)!5! \Rightarrow 4\cdot 3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$

g) they are to be arranged as couples around the dinner table?

5 couples then within each couple $\Rightarrow (n-1)!\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1 \Rightarrow$

$(5-1)!2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1 \Rightarrow 4\cdot 3\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1$

h) only the host and hostess are to sit together?

1 couple then within the couple and 8 singles $\Rightarrow (n-1)!\cdot 2\cdot 1 \Rightarrow$

$(9-1)!2\cdot 1 \Rightarrow 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 2\cdot 1$

5. In how many ways can:

a) 5 different keys be arranged on a key chain? $\frac{(n-1)!}{2} \Rightarrow \frac{(5-1)!}{2} \Rightarrow \frac{4\cdot 3\cdot 2\cdot 1}{2}$

b) 12 different keys be arranged on a key chain? $\frac{(n-1)!}{2} \Rightarrow \frac{(12-1)!}{2} \Rightarrow \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2}$

c) 8 different coloured beads be arranged on a necklace? $\frac{(n-1)!}{2} \Rightarrow \frac{(8-1)!}{2} \Rightarrow \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2}$

d) 6 different charms be arranged on a charm bracelet? $\frac{(n-1)!}{2} \Rightarrow \frac{(6-1)!}{2} \Rightarrow \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{2}$

6. In how many ways can:

a) 4 red beads, 6 blue beads and 4 green beads be arranged on a necklace (only difference in beads is colour)?

$\frac{(n-1)!}{2} \Rightarrow$ must considered repetition of elements $\Rightarrow \frac{(14-1)!}{2\cdot 4!6!4!} \Rightarrow$

$\frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 4\cdot 3\cdot 2\cdot 1}$

b) 3 "c" master keys, 2 "a" master keys and 7 "f" master keys are to be arranged on a key ring?

$\frac{(n-1)!}{2} \Rightarrow$ must considered repetition of elements $\Rightarrow \frac{(12-1)!}{2\cdot 3!2!7!} \Rightarrow$

$\frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 3\cdot 2\cdot 1\cdot 2\cdot 1\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$