PERMUTATIONS

Permutations involving:

1. Alphabet (vowels/consonants)
2. Numbers (focus on zeros)
3. People
4. Items (beads, flags, etc.)

1. How many different 5 letter arrangements can be made from:
   a) vowels $v \cdot v \cdot v \cdot v \cdot v \Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
   b) consonants $c \cdot c \cdot c \cdot c \cdot c \Rightarrow 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17$
   c) first two are vowels and the last three are consonants $v \cdot v \cdot c \cdot c \cdot c \Rightarrow 5 \cdot 4 \cdot 21 \cdot 20 \cdot 19$
   d) the first, third and fifth are consonants and the second and fourth are vowels
   $c \cdot v \cdot c \cdot v \cdot c \Rightarrow 21 \cdot 5 \cdot 20 \cdot 4 \cdot 19$
   e) the first, third and fifth must be consonants and the remaining can be any other letter $c \cdot a \cdot c \cdot a \cdot c \Rightarrow 21 \cdot 23 \cdot 20 \cdot 22 \cdot 19$
   f) the first letter must be "a" and the fourth must be "t" $"a" \cdot a \cdot a \cdot "t" \cdot a \Rightarrow 1 \cdot 24 \cdot 23 \cdot 1 \cdot 22$
   g) the word must end with the letter "z" $a \cdot a \cdot a \cdot a \cdot "z" \Rightarrow 25 \cdot 24 \cdot 23 \cdot 22 \cdot 1$
   h) the word contains exactly 2 vowels that must always be together
   $v \cdot v \cdot c \cdot c \cdot c \Rightarrow 5 \cdot 4 \cdot 23 \cdot 22 \cdot 21 \cdot 4(arrangements)$
   i) the word is made of consonants from the first half of the alphabet
   $c \cdot c \cdot c \cdot c \cdot c \Rightarrow 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

2. Using the digits 2, 3, 6, 8 and 9,
   a) how many three digit can be formed? $5 \cdot 4 \cdot 3$
   b) how many four digit numbers can be formed ? $5 \cdot 4 \cdot 3 \cdot 2$
   c) how many three digit numbers are odd? $\_ \cdot \_ \cdot o \Rightarrow 4 \cdot 3 \cdot 2$
   d) how many three digit numbers are even? $\_ \cdot \_ \cdot e \Rightarrow 4 \cdot 3 \cdot 3$

3. Using the digits 0, 1, 2,... 9 (note: cannot start a number with a "0")
   a) how many five digit numbers can be formed?
   $\_ \cdot \_ \cdot \_ \cdot \_ \cdot 0 + \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \Rightarrow 9 \cdot 8 \cdot 7 \cdot 6 \cdot 1 + 9 \cdot 9 \cdot 8 \cdot 7 \cdot 6$
   b) how many four digit numbers can be formed that are odd?
   $\_ \cdot \_ \cdot \_ \cdot o \Rightarrow 8 \cdot 8 \cdot 7 \cdot 5$
   c) how many six digit numbers can be formed that are divisible by 2?
   $\_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot 0 + \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot (2,4,6,8) \Rightarrow 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 1 + 8 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$
   d) how many four digit numbers can be formed that are divisible by five?
   $\_ \cdot \_ \cdot \_ \cdot 0 + \_ \cdot \_ \cdot \_ \cdot 5 \Rightarrow 9 \cdot 8 \cdot 7 \cdot 1 + 8 \cdot 8 \cdot 7 \cdot 1$
   e) how many 6 digit numbers can be formed in which the digits 3, 4 and 5 must be together in the number and must exist in the number in that order?

"3"·"4"·"5"·_·_·0+"3"·"4"·"5"·_·_·_+_·"3"·"4"·"5"·_·_ ⇒

$1\cdot1\cdot1\cdot6\cdot5\cdot1\cdot3(arrangements)+1\cdot1\cdot1\cdot7\cdot6\cdot5(1\ arrangements)+6\cdot1\cdot1\cdot1\cdot6\cdot5\cdot3(arrangements)$

f) how many five digit numbers can be formed in which the digits 6 and 7 must be together in the number?

"6/7"·"7/6"·_·_·0+"6/7"·"7/6"·_·_·_+_·"6/7"·"7/6"·_·_ ⇒

$2\cdot1\cdot7\cdot6\cdot1\cdot3(arrangements)+2\cdot1\cdot8\cdot7\cdot6(1\ arrangements)+7\cdot2\cdot1\cdot6\cdot5\cdot3(arrangements)$

4. Eight people attend a concert and sit in the front row which has exactly 8 seats:

a) in how many ways can these people be seated? $8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

b) in how many ways can they be seated if a certain person must sit in the left aisle seat? $1\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\Rightarrow1\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

c) in how many ways can they be seated if a certain person must sit the left aisle seat while another person must sit in the right aisle seat?

$1\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\cdot1\Rightarrow1\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot1$

d) in how many ways can they be seated in two people must sit in the aisle seats?

$2\ choices\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\cdot1\ choice\Rightarrow2\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot1$

e) in how many ways can they be seated if two people must sit together?

$2\ choices\cdot1\ choice\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\Rightarrow2\cdot1\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot7(arrangements)$

f) in how many ways can they be seated if four people must sit together?

$4\ choices\cdot3\ choices\cdot2\ choices\cdot1\ choice\cdot\_\cdot\_\cdot\_\cdot\_\Rightarrow$

$4\cdot3\cdot2\cdot1\cdot4\cdot3\cdot2\cdot1\cdot5(arrangements)$

g) in how many ways can they be seated in five people must be seated together?

$5\ choices\cdot4\ choices\cdot3\ choices\cdot2\ choices\cdot1\ choice\cdot\_\cdot\_\cdot\_\Rightarrow$

$5\cdot4\cdot3\cdot2\cdot1\cdot3\cdot2\cdot1\cdot4(arrangements)$

5. Six people (3 males and 3 female) attend a show and sit in six available seats on the left aisle:

a) in how many ways can these six people be seated? $6\cdot5\cdot4\cdot3\cdot2\cdot1$

b) in how many ways can they be seated so that a boy sits in the aisle seat?

$b\cdot\_\cdot\_\cdot\_\cdot\_\cdot\_\Rightarrow6\cdot5\cdot4\cdot3\cdot2\cdot1$

c) in how many ways can they be seated so that male and females alternate with a female at the aisle seat? $f\cdot m\cdot f\cdot m\cdot f\cdot m\Rightarrow3\cdot3\cdot2\cdot2\cdot1\cdot1$

d) in how many ways can they be seated so that male and female alternate?

$f\cdot m\cdot f\cdot m\cdot f\cdot m$ or $m\cdot f\cdot m\cdot f\cdot m\cdot f\Rightarrow3\cdot3\cdot2\cdot2\cdot1\cdot1\cdot2(arrangements)$

6. Five texts, an algebra, a science, a social, a French, and an English are to be arranged on a shelf

a) in how many ways can these books be arranged on a shelf? $5\cdot4\cdot3\cdot2\cdot1$

b) in how many ways can they be arranged so that they are in alphabetic order?

$Al\cdot En\cdot Fr\cdot Sc\cdot So\Rightarrow1\cdot1\cdot1\cdot1\cdot1$

c) in how many ways can they be arranged so that the French text is on the left?

$Fr \cdot \_ \cdot \_ \cdot \_ \cdot \_ \Rightarrow 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

d) in how many ways can they be arranged so that the algebra and science are together all the time? $A/S \cdot S/A \cdot \_ \cdot \_ \cdot \_ \Rightarrow 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 4(arrangments)$

e) in how many ways can they be arranged so that the social text is on the right and the algebra text is on the left? $Al \cdot \_ \cdot \_ \cdot \_ \cdot So \Rightarrow 1 \cdot 3 \cdot 2 \cdot 1 \cdot 1$

7. Five algebra texts, all different, three science texts, all different, and four social texts, all different, are to be arranged on a shelf

a) in how many ways can the texts be arranged on a shelf?

$12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

b) in how many ways can they be arranged in the order of algebra, science and social?

$Al \cdot Sc \cdot So \Rightarrow$

$3 \cdot 2 \cdot 1 \cdot (\text{arrangements within each subject}) \Rightarrow 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

c) in how many ways can they be arranged so that all the social texts will be on the left? $So \cdot So \cdot So \cdot So \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \Rightarrow 4 \cdot 3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

d) in how many ways can they be arranged so that the categories of texts will remain together?

$\text{Arrangement of categories} \cdot \text{arrangements within categories} \Rightarrow$

$3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$