Linear Functions

1. Determine the following information:

   a) slope and y-intercept of the equation –5x + 2y = 7

   $2y = 5x + 7 \Rightarrow y = \frac{5}{2}x + \frac{7}{2} \Rightarrow m = \frac{5}{2}, b = \frac{7}{2}$

   b) the slope of the line segment joining the points (-7, 3) and (4, 8)

   $m = \frac{y_2 - y_2}{x_2 - x_1} \Rightarrow m = \frac{3-8}{-7-4} \Rightarrow m = \frac{-5}{-11} \Rightarrow \frac{5}{11}$

   c) the midpoint of the line segment defined by having endpoints of (3, -8) and (11, 4)

   $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \Rightarrow M\left(\frac{3+11}{2}, \frac{-8+4}{2}\right) \Rightarrow M\left(\frac{14}{2}, \frac{-4}{2}\right) \Rightarrow M(7, -2)$

   d) the distance between the points (-4, 7) and (-3, -9)

   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Rightarrow d = \sqrt{(-4-(-3))^2 + (7-(-9))^2} \Rightarrow d = \sqrt{(-1)^2 + (16)^2} \Rightarrow d = \sqrt{257}$

   e) the slope of a line that is parallel to the line with equation 4x – 7y = 11

   $-7y = -4x + 11 \Rightarrow y = \frac{-4}{-7}x + \frac{11}{-7} \Rightarrow m = \frac{4}{7} \Rightarrow m_2 = \frac{4}{7}$

   f) the slope of the line perpendicular to another line that has a slope of 5/7

   $m_1 = \frac{5}{7}, m_2 = -\frac{7}{5}$

2. Determine the equation of the line given the following information:

   a) m = -5 and b = 3

   $y = mx + b$

   $y = -5x + 3$

   b) m = -3/4, b = 2

   $y = mx + b$

   $y = -\frac{3}{4}x + 2 \Rightarrow 4 \cdot y = 4 \cdot \left(-\frac{3}{4}\right)x + 4 \cdot 2 \Rightarrow$

   $4y = -3x + 8$

   c) m = 5/7 passing through (0, -4)

   $(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$

   $(y - (-4)) = \frac{5}{7}(x - 0) \Rightarrow$

   $7 \cdot (y - (-4)) = 7 \cdot \frac{5}{7}(x - 0) \Rightarrow$

   $7y + 28 = 5x \Rightarrow$

   $7y = 5x - 28$

   d) m = -3 passing through (-3, 4)

   $(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$

   $(y - 4) = -3(x - (-3)) \Rightarrow$

   $y - 4 = -3x - 9 \Rightarrow$

   $y = -3x - 5$

e) m = -5/3 passing through (-2, -6)

$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$$
$$(y - (-6)) = -\frac{5}{3}(x - (-2)) \Rightarrow$$
$$3 \cdot (y - (-6)) = 3 \cdot -\frac{5}{3}(x - (-2)) \Rightarrow$$
$$3y + 18 = -5x - 10 \Rightarrow$$
$$3y = -5x - 28$$

f) passing through the points (-3, 5) and (-1, 6)

$$m = \frac{y_2 - y_2}{x_2 - x_1} \Rightarrow m = \frac{5-6}{-3-1} \Rightarrow m = \frac{-1}{-4} \Rightarrow m = \frac{1}{4}$$
$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$$
$$(y - 5) = \frac{1}{4}(x - (-3)) \Rightarrow$$
$$4 \cdot (y - 5) = 4 \cdot \frac{1}{4}(x - (-3)) \Rightarrow$$
$$4y - 20 = x + 12 \Rightarrow$$
$$4y = x + 32$$

g) through (5, 2) parallel to 3x – 2y = 6

$$-2y = -3x + 6 \Rightarrow y = \frac{-3}{-2}x + \frac{6}{-2} \Rightarrow$$
$$m_1 = \frac{3}{2}, m_2 = \frac{3}{2}$$
$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$$
$$(y - 2) = \frac{3}{2}(x - 5) \Rightarrow$$
$$2 \cdot (y - 2) = 2 \cdot \frac{3}{2}(x - 5) \Rightarrow$$
$$2y - 4 = 3x - 15 \Rightarrow 2y = 3x - 11$$

h) through (-3, 5) perpendicular to –4x + y = 6

$$y = 4x + 6 \Rightarrow m_1 = 4, m_2 = -\frac{1}{4}$$
$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$$
$$(y - 5) = -\frac{1}{4}(x - (-3)) \Rightarrow$$
$$4 \cdot (y - 5) = 4 \cdot -\frac{1}{4}(x - (-3)) \Rightarrow$$
$$4y - 20 = -x - 3 \Rightarrow 4y = -x + 17$$

i) through the point (3, -7) and parallel to the x-axis

since parallel to x-axis

m = 0, ∴ horizontal line

y = -7

j) through the point (-4, -6) and perpendicular to the x-axis

since perpendicular to x-axis

m = no slope, ∴ vertical line

$x = -4$

k) thorough (3, -2) parallel to the line passing through (4, 8) and (6, 16)

$$m = \frac{y_2 - y_2}{x_2 - x_1} = \frac{8-16}{4-6} = \frac{-8}{-2} = 4 \Rightarrow$$
$$m_1 = 4, m_2 = 4$$
$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$$
$$(y - (-2)) = 4(x - 3) \Rightarrow$$
$$y + 2 = 4x - 12 \Rightarrow y = 4x - 14$$

l) through (-5, 1) perpendicular to the line passing through (-5, 2) and (5, 6)

$$m = \frac{y_2 - y_2}{x_2 - x_1} = \frac{2-6}{-5-5} = \frac{-4}{-10} = \frac{4}{10} = \frac{2}{5} \Rightarrow$$
$$m_1 = \frac{2}{5}, m_2 = -\frac{5}{2}$$
$$(y_2 - y_1) = m(x_2 - x_1) \Rightarrow$$
$$(y - 1) = -\frac{5}{2}(x - (-5)) \Rightarrow$$
$$2 \cdot (y - 1) = \cdot 2 - \frac{5}{2}(x - (-5)) \Rightarrow$$
$$2y - 2 = -5x - 25 \Rightarrow 2y = -5x - 27$$

m) perpendicular bisector of the line segment defined by the endpoints (3, 2) and (-9, 10)

$$m=\frac{y_2-y_2}{x_2-x_1}=\frac{2-10}{3-(-9)}=\frac{-8}{12}=-\frac{2}{3}\Rightarrow$$

$$m_1=-\frac{2}{3}, m_2=\frac{3}{2}$$

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{3+(-9)}{2},\frac{2+10}{2}\right)=(-3,6)$$

$$(y_2-y_1)=m(x_2-x_1)\Rightarrow$$

$$(y-6)=\frac{3}{2}(x-(-3))\Rightarrow$$

$$2\cdot(y-6)=2\cdot\frac{3}{2}(x-(-3))\Rightarrow$$

$$2y-12=3x+9\Rightarrow 2y=3x+21$$