COMBINATIONS (from several sets)

1. In how many ways can 3 math books and 2 science books be selected from a set of 12 math books and 9 science books, all different?

$$\frac{12\cdot 11\cdot 10}{3!}\cdot\frac{9\cdot 8}{2!}=\frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1}\cdot\frac{9\cdot 8}{2\cdot 1}=220\cdot 36$$

2. In how many ways can a committee be formed containing three girls and two boys from a group containing ten girls and twelve boys?

$$\frac{10\cdot 9\cdot 8}{3!}\cdot\frac{12\cdot 11}{2!}=\frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}\cdot\frac{12\cdot 11}{2\cdot 1}=120\cdot 66$$

3. In how many ways can a person select a path from A to B and a path from B to C if there are nine paths to chose from A to B and five from B to C?

$$\frac{9}{1!}\cdot\frac{5}{1!}=9\cdot 5$$

4. In how many ways can a student select 4 short answer and three long answer questions on a test from 6 short answer and 5 long answer?

$$\frac{6\cdot 5\cdot 4\cdot 3}{4!}\cdot\frac{5\cdot 4\cdot 3}{3!}=\frac{6\cdot 5\cdot 4\cdot 3}{4\cdot 3\cdot 2\cdot 1}\cdot\frac{5\cdot 4\cdot 3}{3\cdot 2\cdot 1}=15\cdot 10$$

5. In how many ways can a ten card be dealt so that it contains exactly five hearts and 5 spades?

$$\frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5!}\cdot\frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5!}=\frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5\cdot 4\cdot 3\cdot 2\cdot 1}=252\cdot 252$$

6. How many 12 card hands can be dealt that contain exactly 8 spades?

$$\frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6}{8!}\cdot\frac{39\cdot 38\cdot 37\cdot 36}{4!}=\frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6}{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{39\cdot 38\cdot 37\cdot 36}{4\cdot 3\cdot 2\cdot 1}=$$
$$1{,}287\cdot 82{,}251$$

7. How many 13 card hands can be dealt that containing exactly 3 queens, 2 kings, 2 aces and 3 nines?

$$\frac{4\cdot 3\cdot 2}{3!}\cdot\frac{4\cdot 3}{2!}\cdot\frac{4\cdot 3}{2!}\cdot\frac{4\cdot 3\cdot 2}{3!}\cdot\frac{40\cdot 39\cdot 38}{3!}=\frac{4\cdot 3\cdot 2}{3\cdot 2\cdot 1}\cdot\frac{4\cdot 3}{2\cdot 1}\cdot\frac{4\cdot 3}{2\cdot 1}\cdot\frac{4\cdot 3\cdot 2}{3\cdot 2\cdot 1}\cdot\frac{40\cdot 39\cdot 38}{3\cdot 2\cdot 2}=$$
$$4\cdot 6\cdot 6\cdot 4\cdot 9{,}880$$

8. In how many ways can a person select a pair of pants from six pants, a pair of shoes from 3 pairs of shoes, and a hat from 10 hats?

$$\frac{6}{1!}\cdot\frac{3}{1!}\cdot\frac{10}{1!}=6\cdot 3\cdot 10$$

9. In how many ways can you select 3 consonants and 4 vowels from the letters in the alphabet?

$$\frac{21\cdot 20\cdot 19}{3!}\cdot\frac{5\cdot 4\cdot 3\cdot 2}{4!}=\frac{21\cdot 20\cdot 19}{3\cdot 2\cdot 1}\cdot\frac{5\cdot 4\cdot 3\cdot 2}{4\cdot 3\cdot 2\cdot 1}=1{,}330\cdot 5$$

10. In how many ways can you select 2 even and three odd digits from the digits 0, 1, 2, ... , 9?

$$\frac{5\cdot 4}{2!}\cdot\frac{5\cdot 4\cdot 3}{3!}=\frac{5\cdot 4}{2\cdot 1}\cdot\frac{5\cdot 4\cdot 3}{3\cdot 2\cdot 1}=10\cdot 10$$

11. In how many ways can you select 2 vowels and 2 consonants from the word "numbers"?

$$\frac{2\cdot 1}{2!}\cdot\frac{5\cdot 4}{2!}=\frac{2\cdot 1}{2\cdot 1}\cdot\frac{5\cdot 4}{2\cdot 1}=1\cdot 10$$

12. In how many ways can a male leading role, a female leading role, 2 supporting males and 3 supporting females be selected from a group containing nine females and 12 males?

$$\frac{12}{1!}\cdot\frac{9}{1!}\cdot\frac{11\cdot 10}{2!}\cdot\frac{8\cdot 7\cdot 6}{3!}=\frac{12}{1}\cdot\frac{9}{1}\cdot\frac{11\cdot 10}{2\cdot 1}\cdot\frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}=12\cdot 9\cdot 55\cdot 56$$

13. A bridge hand has thirteen cards of each suit. How many thirteen-card hands having exactly eight clubs can be dealt? How many hands having exactly 2 hearts, 5 clubs, 4 spades and 2 diamonds are there?

a)
$$\frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6}{8!}\cdot\frac{39\cdot 38\cdot 37\cdot 36\cdot 35}{5!}=$$
$$\frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6}{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{39\cdot 38\cdot 37\cdot 36\cdot 35}{5\cdot 4\cdot 3\cdot 2\cdot 1}=1{,}287\cdot 575{,}757$$

b)
$$\frac{13\cdot 12}{2!}\cdot\frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5!}\cdot\frac{13\cdot 12\cdot 11\cdot 10}{4!}\cdot\frac{13\cdot 12}{2!}=$$
$$\frac{13\cdot 12}{2\cdot 1}\cdot\frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{13\cdot 12\cdot 11\cdot 10}{4\cdot 3\cdot 2\cdot 1}\cdot\frac{13\cdot 12}{2\cdot 1}=78\cdot 1{,}287\cdot 715\cdot 78$$

14. From a group of 6 men and eight women, how many committees of 3 men and 2 woman can be formed?

$$\frac{6\cdot 5\cdot 4}{3!}\cdot\frac{8\cdot 7}{2!}=\frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1}\cdot\frac{8\cdot 7}{2\cdot 1}=20\cdot 28$$

15. A bag contains 4 red, 6 white and five blue marbles. How many ways can 2 red, 3 white and 3 blue be chosen?

$$\frac{4\cdot 3}{2!}\cdot\frac{6\cdot 5\cdot 4}{3!}\cdot\frac{5\cdot 4\cdot 3}{3!}=\frac{4\cdot 3}{2\cdot 1}\cdot\frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1}\cdot\frac{5\cdot 4\cdot 3}{3\cdot 2\cdot 1}=6\cdot 20\cdot 10$$

16. In how many ways can you select three letters from the word “study” and two letters from the word “charge”?

$$\frac{5\cdot 4\cdot 3}{3!}\cdot\frac{6\cdot 5}{2!}=\frac{5\cdot 4\cdot 3}{3\cdot 2\cdot 1}\cdot\frac{6\cdot 5}{2\cdot 1}=10\cdot 15$$