Rational Expressions

1. For each question determine:
   a) the values of x for which the rational expression is equal to 0
   b) the values of x for which the expression is undefined

   For an expression to have a value of "0" the numerator must equal "0". Therefore, the numerator must be factored and each factor equated to "0" and then solved.
   For an expression to be undefined the denominator must equal "0". Therefore, the denominator must be factored and each factor equated to "0" and then solved.

$$\frac{(x-3)(x+2)}{(x-6)(x+1)}$$

1. $a)\left(x-3\right)=0 \Rightarrow x=3$ and $\left(x+2\right)=0 \Rightarrow x=-2$
   $b)\left(x-6\right)=0 \Rightarrow x=6$ and $\left(x+1\right)=0 \Rightarrow x=-1$

$$\frac{2x^2+3x+1}{x^2-9} \Rightarrow \frac{(2x+1)(x+1)}{(x+3)(x-3)}$$

2. $a)\left(2x+1\right)=0 \Rightarrow x=-1/2$ and $\left(x+1\right)=0 \Rightarrow x=-1$
   $b)\left(x+3\right)=0 \Rightarrow x=-3$ and $\left(x-3\right)=0 \Rightarrow x=3$

$$\frac{x^2-25}{x^2+8x+15} \Rightarrow \frac{(x \not+ 5)(x-5)}{(x+3)(x \not+ 5)}$$

3. $a)(x-5)=0 \Rightarrow x=5$
   $b)(x+3)=0 \Rightarrow x=-3$

2. Simplify the following expressions

1. $\frac{25x^3y^2}{15xy^4}=\frac{5x^2}{3y^2}$

2. $\frac{3x^2-27}{4x-12}=\frac{3(x+3)\cancel{(x-3)}}{4\cancel{(x-3)}}=\frac{3(x+3)}{4}$

3. $\frac{x^2-6x-27}{x^2-2x-15}=\frac{(x-9)\cancel{(x+3)}}{(x-5)\cancel{(x+3)}}=\frac{(x-9)}{(x-5)}$

4. $\frac{3x^2-10x+8}{3x^2-x-4}=\frac{\cancel{(3x-4)}(x-2)}{\cancel{(3x-4)}(x+1)}=\frac{(x-2)}{(x+1)}$

5. $\frac{2x^2+11x+12}{x^3+x^2-12x}=\frac{(2x+3)\cancel{(x+4)}}{x\cancel{(x+4)}(x-3)}=\frac{(2x+3)}{x(x-3)}$

3. Multiplication and division

1. $\dfrac{3x^3}{2y^2}\cdot\dfrac{8y^4}{27x^2}=\dfrac{\cancel{3}\cdot x^3\cdot\cancel{2}\cdot 2\cdot 2\cdot y^4}{\cancel{2}\cdot y^2\cdot\cancel{3}\cdot 3\cdot 3\cdot x^2}=\dfrac{2\cdot 2\cdot x\cdot y^2}{3\cdot 3}$

2. $\left(\dfrac{16x^3y^2}{25ab^5}\right)\div\left(\dfrac{24xy^3}{15a^3b^2}\right)=\left(\dfrac{16x^3y^2}{25ab^5}\right)\cdot\left(\dfrac{15a^3b^2}{24xy^3}\right)=\dfrac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot 2\cdot x^3\cdot y^2\cdot\cancel{3}\cdot\cancel{5}\cdot a^3\cdot b^2}{\cancel{5}\cdot 5\cdot a\cdot b^5\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot x\cdot y^3}=\dfrac{2x^2a^2}{5yb^3}$

3. $\dfrac{x^2-16}{x^2}\cdot\dfrac{x^2-4x}{x^2-x-12}=\dfrac{\cancel{(x+4)}(x-4)\cancel{x}(x-4)}{x^{\cancel{2}}\cancel{(x+4)}(x-3)}=\dfrac{(x-4)(x-4)}{x(x-3)}$

4. $\dfrac{x^2-2x-35}{2x^3-3x^2}\cdot\dfrac{4x^3-9x}{7x-49}=\dfrac{\cancel{(x-7)}(x+5)\cdot\cancel{x}(2x+3)\cancel{(2x-3)}}{x^{\cancel{2}}\cancel{(2x-3)}\cdot 7\cancel{(x-7)}}=\dfrac{(x+5)(2x+3)}{7x}$

5. $\dfrac{x^2-16}{x^2-10x+25}\div\dfrac{3x-12}{x^2-3x-10}=\dfrac{x^2-16}{x^2-10x+25}\cdot\dfrac{x^2-3x-10}{3x-12}=$
$\dfrac{(x+4)\cancel{(x-4)}\cdot\cancel{(x-5)}(x+2)}{\cancel{(x-5)}(x-5)\cdot 3\cancel{(x-4)}}=\dfrac{(x+4)(x+2)}{(x-5)\cdot 3}$

6. $\dfrac{x^3+4x}{x^2-16}\div\dfrac{x^2+8x+15}{x^2+x-20}=\dfrac{x^3+4x}{x^2-16}\cdot\dfrac{x^2+x-20}{x^2+8x+15}=$
$\dfrac{x(x^2+4)\cdot\cancel{(x+5)}\cancel{(x-4)}}{(x+4)\cancel{(x-4)}\cancel{(x+5)}(x+3)}=\dfrac{x(x^2+4)}{(x+4)(x+3)}$

7. $\dfrac{x^2-36}{x^2-8x+16}\div\dfrac{3x-18}{x^2-x-12}=\dfrac{x^2-36}{x^2-8x+16}\cdot\dfrac{x^2-x-12}{3x-18}=$
$\dfrac{(x+6)\cancel{(x-6)}\cdot\cancel{(x-4)}(x+3)}{(x-4)\cancel{(x-4)}\cdot 3\cancel{(x-6)}}=\dfrac{(x+6)(x+3)}{(x-4)3}$

4. Addition and subtraction

1. $\dfrac{3+x}{x}+\dfrac{4}{x}=\dfrac{3+x+4}{x}=\dfrac{(x+7)}{x}$

2. $\dfrac{2x^2+5x-9}{x-5}+\dfrac{x^2-19x+4}{x-5}=\dfrac{2x^2+5x-9+x^2-19x+4}{(x-5)}=\dfrac{3x^2-14x-5}{(x-5)}=$
$\dfrac{(3x+1)\cancel{(x-5)}}{\cancel{(x-5)}}=(3x+1)$

3.

$$\frac{x-2}{x+3}+\frac{x+2}{x-4}=\frac{(x-2)}{(x+3)}+\frac{(x+2)}{(x-4)}=\frac{(x-2)(x-4)+(x+2)(x+3)}{(x+3)(x-4)}=$$

$$\text{LCD} = (x+3)(x-4)$$

$$\frac{x^2-6x+8+x^2+5x+6}{(x+3)(x-4)}=\frac{2x^2-x+14}{(x+3)(x-4)}$$

4.

$$\frac{x^2}{x-5}+\frac{25}{5-x}=\frac{x^2}{(x-5)}+\frac{25}{(5-x)}=\frac{x^2}{(x-5)}+\frac{25}{-1(x-5)}=\frac{x^2}{(x-5)}-\frac{25}{(x-5)}=$$

$$\frac{x^2-25}{(x-5)}=\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}}=(x+5)$$

5.

$$\frac{x-2}{4x+8}-\frac{x+6}{5x+10}=\frac{(x-2)}{(4x+8)}-\frac{(x+6)}{(5x+10)}=\frac{5(x-2)-4(x+6)}{4\cdot 5\cdot(x+2)}=\frac{5x-10-4x-24}{4\cdot 5\cdot(x+2)}=\frac{(x-34)}{4\cdot 5\cdot(x+2)}$$

$$4(x+2) \quad 5(x+2) \quad \text{LCD} = 4\cdot 5\cdot(x+2)$$

6.

$$\frac{1}{2x}+\frac{5x}{x^2-1}+\frac{3}{x+1}=\frac{1}{2x}+\frac{5x}{(x^2-1)}+\frac{3}{(x+1)}=\frac{1\cdot(x+1)(x-1)+5x(2x)+3\cdot 2x\cdot(x-1)}{2x(x+1)(x-1)}=$$

$$(x+1)(x-1) \quad LCD = 2x(x+1)(x-1)$$

$$\frac{x^2-1+10x^2+6x^2-6x}{2x(x+1)(x-1)}=\frac{17x^2-6x-1}{2x(x+1)(x-1)}$$

7.

$$\frac{3x}{x^2-7x+10}-\frac{2x}{x^2-8x+15}=\frac{3x}{(x^2-7x+10)}-\frac{2x}{(x^2-8x+15)}=\frac{3x(x-3)-2x(x-2)}{(x-5)(x-2)(x-3)}=$$

$$(x-5)(x-2) \quad (x-5)(x-3) \quad LCD = (x-5)(x-2)(x-3)$$

$$\frac{3x^2-9x-2x^2-4x}{(x-5)(x-2)(x-3)}=\frac{x^2-13x}{(x-5)(x-2)(x-3)}$$

8.

$$\frac{3x-2}{x^2+2x-24}-\frac{x-3}{x^2-16}=\frac{(3x-2)}{(x^2+2x-24)}-\frac{(x-3)}{(x^2-16)}=\frac{(3x-2)(x+4)-(x-3)(x+6)}{(x+6)(x-4)(x+4)}=$$

$$(x+6)(x-4) \quad (x+4)(x-4) \quad LCD = (x+6)(x-4)(x+4)$$

$$\frac{\left[3x^2+12x-2x-8\right]-\left[x^2+6x-3x-18\right]}{(x+6)(x-4)(x+4)}=\frac{3x^2+12x-2x-8-x^2-6x+3x+18}{(x+6)(x-4)(x+4)}=$$

$$\frac{2x^2+7x+10}{(x+6)(x-4)(x+4)}$$

9.

$$\frac{2}{x+3}-\frac{x}{x-1}+\frac{x^2+2}{x^2+2x-3}=\frac{2}{(x+3)}-\frac{x}{(x-1)}+\frac{(x^2+2)}{(x^2+2x-3)}=\frac{2(x-1)-x(x+3)+(x^2+2)}{(x+3)(x-1)}=$$

$(x+3)(x-1)$ $LCD = (x+3)(x-1)$

$$\frac{2x-2-x^2-3x+x^2+2}{(x+3)(x-1)}=\frac{-x}{(x+3)(x-1)}$$

5. Solve the following equations

1.

$$\frac{2}{5}+\frac{t}{4}=1 \Rightarrow LCD=5\cdot 4 \Rightarrow \cancel{5}\cdot 4\cdot\frac{2}{\cancel{5}}+5\cdot\cancel{4}\cdot\frac{t}{\cancel{4}}=5\cdot 4\cdot 1 \Rightarrow$$

$$8+5t=20 \Rightarrow 5t=12 \Rightarrow t=\frac{12}{5} \Rightarrow \left\{\frac{12}{5}\right\}$$

2.

$$\frac{x+1}{3}-\frac{x+2}{6}=\frac{x+5}{4} \Rightarrow \frac{(x+1)}{3}-\frac{(x+2)}{6}=\frac{(x+5)}{4} \Rightarrow$$

3 $2\cdot 3$ $2\cdot 2 \Rightarrow 2^2$ $LCD = 2^2\cdot 3$

$$2^2\cdot\cancel{3}\cdot\frac{(x+1)}{\cancel{3}}-2^{\cancel{2}}\cdot\cancel{3}\cdot\frac{(x+2)}{\cancel{2}\cdot\cancel{3}}=\cancel{2^2}\cdot 3\cdot\frac{(x+5)}{\cancel{2^2}} \Rightarrow$$

$$4x+4-2x-4=3x+15 \Rightarrow -x=15 \Rightarrow x=-15 \Rightarrow \{-15\}$$

3.

$$\frac{x}{3}+\frac{x}{4}=\frac{7}{2} \Rightarrow 2^2\cdot\cancel{3}\cdot\frac{x}{\cancel{3}}+\cancel{2^2}\cdot 3\cdot\frac{x}{\cancel{2^2}}=2^{\cancel{2}}\cdot 3\cdot\frac{7}{\cancel{2}} \Rightarrow 4x+3x=42 \Rightarrow 7x=42 \Rightarrow x=6 \Rightarrow \{6\}$$

3 2^2 2 $LCD = 2^2\cdot 3$

4.

$$\frac{4}{x-5}+\frac{3}{x+5}=\frac{40}{x^2-25} \Rightarrow \frac{4}{(x-5)}+\frac{3}{(x+5)}=\frac{40}{(x^2-25)} \Rightarrow$$

$(x-5)(x+5)$ $LCD = (x-5)(x+5)$

$$\cancel{(x-5)}(x+5)\cdot\frac{4}{\cancel{(x-5)}}+(x-5)\cancel{(x+5)}\cdot\frac{3}{\cancel{(x+5)}}=\cancel{(x-5)}\cancel{(x+5)}\cdot\frac{40}{\cancel{(x+5)}\cancel{(x-5)}} \Rightarrow$$

$$4x+20+3x-15=40 \Rightarrow 7x=35 \Rightarrow x=5 \Rightarrow \{5\}$$

5.

$$\frac{5}{x-10}+\frac{2}{x-4}=\frac{9}{x^2-14x+40}\Rightarrow\frac{5}{(x-10)}+\frac{2}{(x-4)}=\frac{9}{(x^2-14x+40)}\Rightarrow$$

$(x-10)(x-4)$ $\quad LCD = (x-10)(x-4)$

$$\cancel{(x-10)}(x-4)\cdot\frac{5}{\cancel{(x-10)}}+(x-10)\cancel{(x-4)}\cdot\frac{2}{\cancel{(x-4)}}=\cancel{(x-10)}\cancel{(x-4)}\cdot\frac{9}{\cancel{(x-10)}\cancel{(x-4)}}\Rightarrow$$

$$5x-20+2x-20=9\Rightarrow 7x=49\Rightarrow x=7\Rightarrow\{7\}$$

6.

$$\frac{5x}{x+1}+\frac{4}{x}=9\Rightarrow\frac{5x}{(x+1)}+\frac{4}{x}=9\Rightarrow x\cancel{(x+1)}\cdot\frac{5x}{\cancel{(x+1)}}+\cancel{x}(x+1)\cdot\frac{4}{\cancel{x}}=x(x+1)\cdot 9\Rightarrow$$

$LCD = x(x+1)$

$$5x^2+4x+4=9x^2+9x\Rightarrow -4x^2-5x+4=0\Rightarrow 4x^2+5x-4=0\Rightarrow \text{can not factor}\therefore\{\ \}$$