

Laboratory # 4

Average and Instantaneous Velocity

Teaches:

- average and instantaneous velocity
 - rectilinear motion
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1. Laura lives in Hamilton, New York; her parents live 60 miles away in Syracuse. She recently took a car trip to visit her parents. She left Hamilton at 12:00 p.m. and arrived at her parents' house at 1:30 p.m. Laura claims that she never once exceeded the 55 miles per hour speed limit during the entire trip. In this lab, we shall try to discover whether Laura was telling the truth.

2. Choose **File** \gg **Load** \gg **Utility** to load the file LABS.MTH, then Author  and Plot 

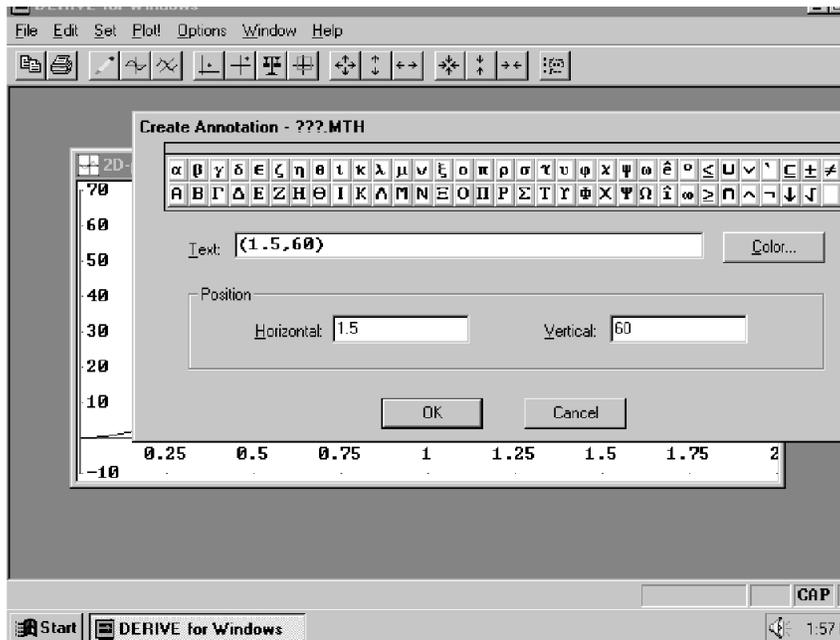
$$S(t) := \text{If} \left(t \leq 1.5, \frac{t^2(36t^2 - 428t + 801)}{9}, 0 \right)$$

Choose **Set** \gg **Range** and set Top=65, Left=0, Bottom=-15, and Right=2. Also turn Trace Mode on (.

3. The graph in front of you is the plot of Laura's trip; time (in hours, starting at 12:00 p.m.) is plotted on the horizontal axis, while distance travelled (in miles) is plotted on the vertical axis. So for example, the graph begins at the origin (0,0) indicating that at noon ($x = 0$) she had travelled 0 miles, and it

ends at $(1.5, 60)$ indicating that $1\frac{1}{2}$ hours later ($x = 1.5$) she had travelled 60 miles. By moving the cross along the graph and reading off the coordinates of the cross from the bottom of the screen, you can estimate how far Laura had travelled at any time x , $0 \leq x \leq 1.5$.

Move the cross to $x = 1.5$, then label this point by clicking the Create Annotation button . Type $(1.5, 60)$ in the text box:

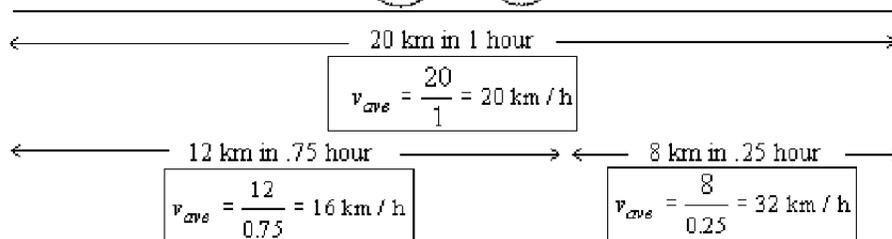


Continuing in this way, label the points corresponding to $x = 0, 0.25, 0.5, 0.75, 1, 1.25,$ and 1.5 .

Definition 4.1 (Average Velocity) *The average velocity of a moving object over a given time interval is defined by*

$$v_{ave} = \frac{\text{total distance travelled during the time interval}}{\text{length of time interval}}$$

For example, if a cyclist rides a 20-kilometer race in 1 hour, then his average velocity is $\frac{20\text{km}}{1\text{h}} = 20 \text{ km/h}$. However, if the distance he has travelled after 45 minutes is 12 km, then his average velocity during the first 45 minutes is $\frac{12\text{km}}{.75\text{h}} = 16 \text{ km/h}$, and his average velocity during the last quarter hour is $\frac{(20-12)\text{km}}{.25\text{h}} = 32 \text{ km/h}$, indicating that he was travelling twice as fast (on average) during the last quarter hour than during the first 45 minutes of the race.



Answer Question #1.

4. In Question #2, you need to compute Laura's average velocity during the first 45 minutes and during the last 45 minutes of her trip. Use the information gathered in Question #1. Copy the graph of Laura's trip into the picture box, and use the Paint program  to draw two lines as indicated.
5. Answer Questions #2, #3, and #4. In each case, copy the graph of Laura's trip in the given picture box, then use Paint to draw the indicated lines. Your answers should indicate that sometime during the second half hour of the trip Laura must have exceeded the speed limit. Was she speeding during the whole half hour, or only for a couple of minutes, perhaps to pass another vehicle? In order to answer these questions, we need to keep calculating her average velocity over smaller and smaller time

intervals.

6. Since computing average velocities can get a bit tedious, I have defined some functions to help you. The function VELOCITY will calculate average velocities for you. In your Algebra window, Author

$$\text{VELOCITY}(S(t), 0, 1.5, 3)$$

and Approximate . You will obtain the vector

$$[33.1111, 57.7777, 29.1111]. \quad (4.1)$$

These numbers represent the average velocities during the first, second, and third half hours of the trip. They should be close to your answers to Worksheet Question #3. To calculate average velocities over 15-minute intervals, you must divide the original time interval $[0, 1.5]$ into six subintervals, each of length $1/4$ hour. To do this, Author

$$\text{VELOCITY}(S(t), 0, 1.5, 6)$$

and Approximate . These should be close to your answers to Question #4. (In general, $\text{VELOCITY}(S(t), a, b, n)$ calculates the average velocities over the time intervals obtained by dividing the interval $[a, b]$ into n equal subintervals.) Answer Worksheet Question #5. [Hint: Save yourself some typing by using the Copy (Ctrl-C) and Paste (Ctrl-V) capabilities of CalcLabs and Derive.]

7. The function VELPLOT will give you a plot of the average velocities. Delete all graphs and annotations. Then Author $\text{VELPLOT}(S(t), 0, 1.5, 3)$, then approximate and Plot. As you can see, the graph you obtain consists of 3 horizontal lines whose heights are given precisely by (4.1). Answer Question #6. Use Trace Mode to approximate the interval during which Laura reached or exceeded the 55 mph speed limit.

The graph that you've obtained with VELPLOT is a graph of **average** velocities over tiny (one-minute) intervals. This is a good approximation of Laura's **instantaneous velocity**.

Definition 4.2 (Instantaneous Velocity) *The instantaneous velocity of a moving object at a given time $t = t_0$ is defined by*

$$v(t_0) = \lim_{\Delta t \rightarrow 0} \frac{\text{distance travelled between times } t = t_0 \text{ and } t = t_0 + \Delta t}{\Delta t}$$

You will learn later on that the instantaneous velocity can be obtained by computing a function called the derivative of $S(t)$. Just for fun, highlight the expression $\frac{t^2(36t^2-428t+801)}{9}$ and click the Derivative button ; then click on Simplify. Plot the resulting expression.

8. Answer Question #7.
9. Author $S(t) := -4t^2 + 10t + 6$. Then Author and Approximate VELOCITY($S(t)$, 0, 3, 3) to obtain average velocities over the three intervals $[0, 1]$, $[1, 2]$, and $[2, 3]$. Answer Question #8.
10. Plot $S(t)$. Choose **Set** \gg **Range** and set Top=15, Left=-0.5, Bottom=-5, and Right=3.5. This should give you a good view of the graph for $0 \leq x \leq 3$. On this graph, time t is measured on the x axis, and the position of the fly is measured on the y axis. Answer Question #9.
11. To obtain an approximate graph of the velocity of the fly, Author, Approximate, and Plot:

VELPLOT($S(t)$, 0, 3, 100)

Dividing the interval $[0, 3]$ into 100 equal subintervals, this graph consists of 100 small horizontal line segments, each at the height of the average velocity of the fly over one of these small subintervals. This graph is an approximation of the **derivative** of $S(t)$, which gives the exact instantaneous velocity of the fly at each instant t . Obtain the derivative of $S(t)$ by highlighting the expression for $S(t)$, clicking the Derivative button , and then clicking Simplify. Plot the resulting expression to verify that it's close to the VELPLOT graph. Answer Question #10.