

Laboratory # 1

Functions and Graphs

Teaches:

- functions – domain, range, x and y intercepts, symmetry, odd, even
 - using Derive to find zeroes
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-

1. When you run CalcLabs, a session of Derive is automatically activated in the top half of your screen.







You will see an Algebra window on the right, where you can enter expressions and perform various algebraic operations. On the left is a graphics window, where you can plot functions. You can make either window active by clicking anywhere inside it. Notice how when you click back and forth between the two windows, the toolbars and menus change.

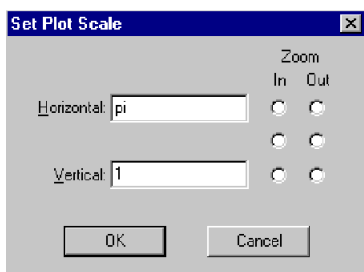
2. Activate the Algebra window, then click Author  to enter the following expression:

$$\sin(x) - \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5}$$

then click OK.



3. Activate the graphics window then click the Plot button  to plot the highlighted expression in the Algebra window. Notice that there's a cross on the screen whose coordinates are given at the bottom. The cross can be moved using the arrow keys. To move it up and down by larger increments, use the Page Up and Page Down keys. To move it left and right by larger increments, hold down the Control key and use the left and right arrow keys. You can also move the cross to any point by clicking on that point with your mouse.
4. Zoom out . This allows you to see more of the graph. If you look at the bottom of the screen, you'll see the x and y scales increase each time you click . The x scale is the number of units that each tick on the x axis represents; likewise for the y scale. To zoom in click , so each time you click  you'll see the scales decrease. You can also use the Scale button  to set your scales manually. Click the Scale button and change the x (horizontal) scale to π (just type pi) and your y (vertical) scale to 1.











Definition 1.1 The x -intercepts of the graph of f are the x coordinates of the points where the graph crosses the x axis. The y -intercept of the graph of f is the y coordinate of the point where the graph crosses the y axis.

5. Can you determine the x and y intercepts from the graph, given that each tick on the x axis represents π units?





Definition 1.2 The **domain** of a function $y = f(x)$ is the set of all x 's for which the function is defined. The **range** of a function $y = f(x)$ is the set of all y 's such that $y = f(x)$ for some x .

6. Since the range is the set of all y values used by the graph, to determine the range of the function that you graphed, you need to determine the y coordinates of the highest and lowest points of the graph. Click on one of the highest points of the graph. Notice how the cross moves to this point. Can you guess what the exact value of the x coordinate of this point is in terms of π ? Use the F3 key to toggle in and out of Trace Mode. When you are in trace mode, the cross changes to a box, and the left and right arrow keys move the box left and right while keeping it always on the graph.
7. To get a more accurate approximation for the coordinates of the highest point, Zoom in on this point ✱ several times. Occasionally, you will need to center the graph on the cross + to keep it in your viewing window. Obtain as accurate an approximation as you can for the y coordinates of the highest and lowest points of the graph. What is the range of this function?
8. You can annotate graphs in Derive. Change your scales back to π and 1, and center the graph at the origin +. In your graphics window, choose **Options** \gg **Points** and select Size Large. In your Algebra window, Author $[\pi, 0]$ (click the π button to enter π into the Authoring box), then Plot. This puts a large dot at the point $(\pi, 0)$. Now choose **Options** \gg **Axes** and turn Labels Off. Now your axes are not labeled. Toggle out of Trace Mode F3 if necessary. Move the cross to where you want to put the label π on the x axis (a little below the dot). Then click the Annotate button ✎. Enter π (either by clicking the π button or by typing Ctrl-p) then click OK. To delete an annotation, click on it and choose **Edit** \gg **Delete Annotation**. Use this technique to place dots on one of the highest points of the graph, and label it with its (x, y) coordinates. Do the same thing with one of the lowest points of the graph.
9. The bottom of your screen is the CalcLabs workbook, which you will complete and hand in to your instructor, either electronically by saving your work in a file in a shared directory designated by your instructor, or by printing out your file and handing in a hard copy of your work.

10. Click anywhere inside the CalcLabs workbook to activate it. Type an answer, simply click inside the text box provided and type. If you need to type mathematical symbols or Greek letters not available on your keyboard, click the Symbols button , then click on the symbol that you want. Type in answers to the four questions in Question #1.
11. The large yellow box is a picture box. To copy the graph from Derive into it, activate Derive's graphics window, then click the Copy Plot Window button . This will put a copy of the window into the clipboard. Then activate CalcLabs, click inside the yellow picture box, then click the Paste button . Alternatively, click inside the picture box with your right mouse button and select Paste.
12. If you wish to further edit your picture, you can activate the Paint program. You can do this in one of three ways:
- Double-click inside the picture box.
 - Click the right mouse button inside the picture box, and select Edit.
 - Click the Edit Picture button .
13. Use Paint to label the x axis at multiples of π . Use the text tool (the large A). Create a text frame by dragging the mouse pointer diagonally, and then click the font, size, and style you want. Greek letters can be obtained with the Symbol font. Click inside the text frame, and then type the text (type p for π). To insert the text into the picture, click outside the text frame. You can drag the text anywhere by selecting it with the Select tool (the dotted rectangle) and then dragging. When you are finished editing, click outside the picture to exit Paint. You will then be given the option of saving any changes that you made. Click Yes to save.
14. One way to evaluate a function at a particular x is to use the Sub button . For example, to evaluate expression #1 at $x = \pi/2$:
- make sure the Algebra window is active;
 - make sure the expression you wish to evaluate is highlighted (i.e. expression #1);
 - click the Sub button .

- (d) highlight the variable for which you wish to make a substitution (in this case x is the only variable) and in the substitution box, type in the value of x , i.e. $\pi/2$ (Ctrl-p for π , or click the π button), then click OK;
- (e) finally, click the Simplify button  to evaluate exactly, or the Approximate button  to find a decimal approximation.



15. In Question #2, you will evaluate the function $f(x) = \sin x - \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5}$ at various values of x . You can copy your answers from Derive to CalcLabs by using standard windows Copy, and Paste commands. Highlight the expression in Derive that you wish to copy, then choose **Edit** \gg **Copy Expressions** (or type Ctrl-C). Then click inside the text box in the CalcLabs workbook, and Click the Paste button  (or type Ctrl-V). Complete Question #2.
16. Delete any annotations in your graph of $f(x)$, turn your axes labels back on, and change your scales both to 1. To examine the graph of $f(x - 1)$, highlight expression #1, click the Sub button  and type $x - 1$ in the substitution box. Then click OK. Active the graphics window and Plot . Also plot $f(x + 2)$.
17. Use either Derive's annotation feature or Paint to clearly label the three graphs. Then copy the graphs into the first picture box in Question #3. Use the Delete Last Plot button  twice to delete the last two plots. Also delete any annotations (**Edit** \gg **Delete Annotation**). Plot $f(2x)$ and $f(x/2)$. Copy these three graphs into the second picture box of Question #3. Make sure they are clearly labeled. Complete Question #3.
18. Answer Question #4.

Definition 1.3 A **zero** or **root** of a function f is any number a such that $f(a) = 0$.

19. In terms of the graph of f , the zeroes are the x - intercepts. So once you Plot a function, you can approximate the zeroes by moving the cross onto the x -intercepts and reading off the x coordinate of the point from the bottom of the screen. You can increase the accuracy of these approximations by zooming in on the zeroes. In Question #5, use this technique to approximate all the zeroes of the given functions correct to three decimal places.

20. To find the zeroes of a polynomial function algebraically, we factor the polynomial and set each factor equal to 0. Author

$$x^3 - 2x^2 - x + 2$$

then choose **Simplify** \gg **Factor** (or type Ctrl-F). Click Factor. What are the zeroes of this function?

Enter these into Question #6.

21. Author

$$x^3 - x^2 + x - 1$$



and factor as above. Since $x^2 + 1 = 0$ has no real solutions, this cubic only has one real zero. Enter it into Question #6.

22. Author and Factor

$$2x^4 + x^3 - 3x^2 + x$$

Two of the zeroes can be found by setting the two linear factors equal to 0. To set a quadratic factor $ax^2 + bx + c$ equal to 0, you need the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Derive can do this for you: highlight the quadratic factor only (click the left mouse button on the subexpression until only the desired subexpression is highlighted) then click the Solve button . Click Simplify. This will give you the exact zeroes of the quadratic. You can obtain decimal approximations by clicking the Approximate button . Complete Question #6. Remember to use Copy (Ctrl-C) and Paste (Ctrl-V) to copy expressions from Derive.

23. Author and Factor

$$16x^4 - 20x^3 + 5x$$

Note that one of the factors is cubic. Formulas for solving cubic equations exist, though they are usually not taught in school like the quadratic formula is. Derive knows how to solve cubics exactly: highlight the cubic factor and click the Solve button as before. Then click Simplify. We are only interested in the real zero (i.e. the one not involving the imaginary number i). Enter the zeroes into Question #7.

24. Finally, Author and Factor

$$2^x - x^2$$

As you can see, this expression cannot be factored. Try solving with the Solve button. If you look at the graph, you can obtain the exact value of two of the three zeroes (plug them in to verify that they are zeroes). What about the third zero? We can't find its exact value algebraically. There are various approximation techniques that we will learn. To have Derive use one of these approximation techniques, highlight the expression $2^x - x^2$, then choose **Solve** \gg **Numerically**. For the upper and lower bounds, you should enter an interval that contains the zero that you are after. Make your interval small enough so that it doesn't contain the other two zeroes. Click Simplify. Complete Question #7.