Calculus Review

A. Functions:

1. Identify the domain and range for the following:

a) $f(x)=\sqrt{3-x^2}$ b) $f(x)=\dfrac{3}{x^2-7x-8}$

2. Determine the equation given:

a) $D=(-\infty,-5)\cup(5,\infty),\quad R=[0,\infty)$

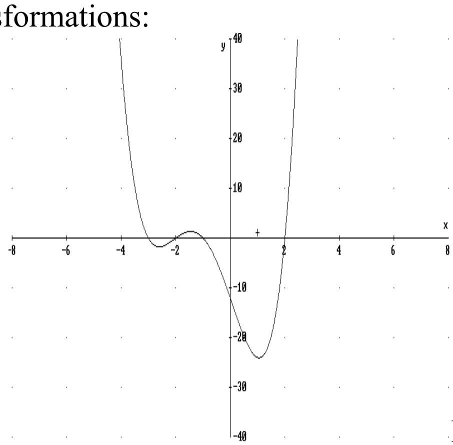
b) $D=(-\infty,-3)\cup(-3,2)\cup(2,\infty),\quad R=(-\infty,0)\cup(0,\infty)$

3. Determine the slope of the tangent line to the curve $y=x^2-3x$. The slope formula must be developed using the slope formula and not differentiation.

4. Given: $f(x)=x^2-5$ and $g(x)=x+1$, determine:

a) $f(-4)$ b) $g(7)$ c) $2f(x)-3g(x)$ d) $(f\circ g)x$

5. Transformations:

Given: 

Determine a) *–f(x),* b) *f(x)* + 1, c) *f(x-1)*

B. Limits

a) General

1. $\lim_{x\to4}\sqrt{x+\sqrt{x}}$ 2. $\lim_{x\to-1}\dfrac{x+1}{x\quad-x}$ 3. $\lim_{x\to0}\dfrac{1-\sqrt{1-x^2}}{x}$

4. $\lim_{x\to-1}\dfrac{x^2-x-2}{x^2+3x+2}$ 5. $\lim_{x\to0^-}\sqrt{-x}$ 6. $\lim_{x\to2}\dfrac{x^2+2x-8}{x^4-16}$

7. $\lim_{x\to\infty}\dfrac{2x^2-x-1}{4x^2+7}$ 8. $\lim_{x\to\infty}\dfrac{5x^3-7x^2}{4x^2+9x-5}$ 9. $\lim_{x\to-3^-}\dfrac{4x+1}{x+3}$

10. $\lim_{x\to0}\dfrac{\sin 5x}{3x}$ 11. $\lim_{x\to0}\dfrac{\sec x}{1-\sin x}$ 12. $\lim_{x\to0}\dfrac{\sin^2 3x}{x^2}$

b) Determine whether each of the following is continuous or discontinuous. If Discontinuous, determine whether a removable discontinuity exists and what it is.

a) $f(x) = -3x^3 + 6x - 7$ b) $f(x) = \dfrac{4}{9 - x^2}$ d) $f(x) = \dfrac{x^2 - 25}{x - 5}$

C. Differentiation

a) General

1. $f(x) = x^3 + 5x + 4$
2. $f(x) = \dfrac{4 - x}{3 + x}$
3. $f(x) = \sqrt{3 - 5x}$
4. $f(x) = x \sin x$
5. $f(x) = \dfrac{x}{\sqrt{9 - 4x}}$
6. $f(x) = \sin(\cos x)$
7. $f(x) = \sin x \cos x$
8. $f(x) = (x^2 - 3)^4 (3x - 7)^5$
9. $f(x) = e^{7x-5}$
10. $f(x) = \ln(x^2 - 5) \cdot 7^{8x-1}$
11. $f(x) = \dfrac{\log_3(x^2 - 5x + 1)}{e^{3x+1}}$
12. $f(x) = \dfrac{1}{\sin(x - \sin x)}$
13. $f(x) = 5x^3 - 2x^2 + 5x - 3$
14. $f(x) = (\ln(3x^2 - 5))^3$
15. $f(x) = 7^{5x^3 - 3x + 6}$
16. $f(x) = \log_9(6x^5 - 7x)$
17. $f(x) = e^{5x-7}$
18. $f(x) = (\cos(6x^3 - 2x + 1))^3$

b) Higher Order

1. $f(x) = 4x^3 - 3x^2 - 18x + 5$ - 3rd order derivative
2. $f(x) = \sin^2 x \cos x$ - 2nd order derivative
3. If $f(x) = (2 - x^2)^6$, find f(0), f'(0), f''(0)

c) Implicit Differentiation

1. $x^2 y + xy^3 = 2$
2. $\ln(x^2 + 1) + 8xy - e^{2y} = 0$
3. $\sqrt[3]{x} - \sqrt{y} = 2$

D. Integration

a) Definite Integral

1. $\int_{-3}^{0} (2x^3 - 3x - 4)dx$
2. $\int_{0}^{1} (5\cos x + 4x)dx$
3. $\int_{1}^{\sqrt{3}} \dfrac{6}{1 + x^2} dx$

b) General

1. $\int 5dx$
2. $\int (3x - 7)dx$
3. $\int \sqrt{x}dx$
4. $\int \sqrt{x} - \dfrac{2}{\sqrt{x}} dx$
5. $\int \left(x + \dfrac{1}{x} \right)^2 dx$
6. $\int \dfrac{1}{x^2 + 36} dx$

7. $\int \frac{x+3}{(x^2+6x)^2} dx$

8. $\int \ln(5x^2-3)x dx$

9. $\int 7^{x^2-7} 2x dx$

10. $\int e^x \sin(e^x) dx$

11. $\int \frac{\sin x}{1+\cos^2 x} dx$

12. $\int \frac{1}{x\sqrt{\ln x}} dx$

13. $\int x^2 \sin 2x dx$

14. $\int \cos x \ln(\sin x) dx$

15. $\int \frac{3x^2-6x+2}{2x^3-3x^2+x} dx$

E. Curve Sketching:

1. $f(x) = x^4 - 6x^2$

2. $f(x) = \frac{1}{x^2(x+3)}$

F. Problem Solving (Related Rates)

1. For $s(t) = t^3 - 3t^2 + 5$ determine a) velocity at t = 2, b) acceleration at t = 2, c) maximum height reached, d) time it takes to reach the ground, e) total distance traveled.
2. Find the equation of a line tangent to the curve $f(x) = 2x^3 - 4x + 1$ at the point having an x-coordinate of -2.
3. What is the slope of the line tangent to the curve $x^3 + 2x^2 y + y = 5$ at the point (-1, 2)
4. A spherical balloon is being inflated at a rate of 10 cubic meters per minute. Find the rate at which the radius is increasing a) when the radius is 5m, b) when the volume is 36 meters cubed.
5. A ladder 8m long is leaning against a wall. The bottom of the ladder is sliding away from the wall at 1.5 m/s. At what rate is thee top of the ladder sliding down the wall at the instant when the bottom of the ladder is 5 meters from the wall?
6. Crushed gravel is being unloaded from a conveyor belt and as it is being poured the gravel forms a conical pile whose base radius is increasing as its height is increasing. If the base radius is increasing at 0.2 m/min and the height is increasing at 0.3 m/min, find the rate at which the volume is increasing?
7. Water is being poured into a conical tank at a rate of 30 cubic meters per minute. If the height and radius at the top of the tank are 12m and 8 meters respectively, find the rate at which the water level is rising at the instant when the height is 4m.
8. Two ships leave port at the same time. Ship a travels west at 20km, while ship B heads south at 35 km. At what rate are the ships separating after one hour?

G. Optimization:

1. A piece of wire 8cm long is cut into two pieces. One piece is bent to form a circle and the other is bent to form a square. How should the wire be cut if the total enclosed area is to be as large as possible?
2. A rectangular field along a straight river is to be divided into 3 smaller fields by one fence parallel to the river and 4 fences perpendicular to the river. Find the maximum area that can be enclose dif 1600m of fencing is available

3. A box with an open top is to be made from a square piece of cardboard, of side length 100cm, by cutting a square from each corner and then folding up the sides. Find the dimensions of the box of largest volume.
4. If the sum of two non-negative numbers is 20, how should the numbers be chosen so that the sum of their squares is a maximum?
5. Find the point on the curve defined by $x^2 - y^2 = 16$ that is closest to the point (0, 2).
6. A can is to be made to hold 3 liters of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

H. Area under a curve.

1. Find the area enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
2. Find the area enclosed by the parabola $y^2 = 4x$ and the line $y = 2x - 4$.
3. Find the volume of a solid obtained by rotating the region bounded by $y = \sqrt[3]{x}$, $y = 8$ and x = 0 around the y-axis.