Limits

1. $\lim_{x\to -2}(3x^3-2x+7)=3(-2)^3-2(-2)+7=-24+4+7=-13$

2. $\lim_{x\to 4}(5x^2-9x-8)=5(4)^2-9(4)-8=80-36-8=36$

3. $\lim_{x\to \sqrt{2}}(x^2+3)(x-4)=\left((\sqrt{2})^2+3\right)\left(\sqrt{2}-4\right)=5\left(\sqrt{2}-4\right)=5\sqrt{2}-20$

4. $\lim_{x\to -3}(3x+4)(7x-9)=\left(3(-3)+4\right)\left(7(-3)-9\right)=\left(-5\right)\left(-30\right)=150$

5. $\lim_{x\to 4}\sqrt[3]{x^2-5x-4}=\sqrt[3]{4^2-5(4)-4}=\sqrt[3]{-8}=-2$

6. $\lim_{x\to -2}\sqrt{x^4-4x+1}=\sqrt{(-2)^4-4(-2)+1}=\sqrt{16+8+1}=5$

7. $\lim_{x\to \frac{1}{2}}\dfrac{4x^2-6x+3}{16x^3+8x-7}=\dfrac{4\left(\frac{1}{2}\right)^2-6\left(\frac{1}{2}\right)+3}{16\left(\frac{1}{2}\right)^3+8\left(\frac{1}{2}\right)-7}=\dfrac{4\cdot\frac{1}{4}-6\cdot\frac{1}{3}+3}{16\cdot\frac{1}{16}+8\cdot\frac{1}{2}-7}=\dfrac{1-2+3}{1+4-7}=\dfrac{2}{-2}=-1$

8. $\lim_{x\to 15}\sqrt{2}=\sqrt{2}$

9. $\lim_{x\to -3}\dfrac{x+3}{\frac{1}{x}+\frac{1}{3}}=\lim_{x\to -3}\dfrac{x+3}{\frac{3+x}{3x}}=\lim_{x\to -3}\dfrac{3x(x+3)}{(3+x)}=3(-3)=-9$

10. $\lim_{x\to 2}\dfrac{x-2}{x^3-8}=\lim_{x\to 2}\dfrac{\left(x-2\right)}{(x-2)(x^2+2x+4)}=\dfrac{1}{2^2+2(2)+4}=\dfrac{1}{12}$

11. $\lim_{x\to 2}\dfrac{x^2-x-2}{(x-2)^2}=\lim_{x\to 2}\dfrac{(x-2)(x+1)}{(x-2)^2}=\lim_{x\to 2}\dfrac{(x+1)}{(x-2)}=\dfrac{2+1}{2-2}=\textit{undefined}$

12. $\lim_{x\to 16}\dfrac{x-16}{\sqrt{x}-4}=\lim_{x\to 16}\dfrac{(\sqrt{x}-4)(\sqrt{x}+4)}{\left(\sqrt{x}-4\right)}=\sqrt{16}+4=8$

13. $\lim_{x\to -2}\frac{x^3+8}{x^4-16}=\lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{(x^2+4)(x+2)(x-2)}=\frac{(-2)^2-2(-2)+4}{\left((-2)^2+4\right)(-2-2)}=\frac{4+4+4}{8\cdot -4}=$
$\frac{12}{-32}=-\frac{3}{8}$

14. $\lim_{x\to 1}\left(\frac{x^2}{x-1}-\frac{1}{x-1}\right)=\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}\frac{(x+1)(x-1)}{(x-1)}=1+1=2$

15. $\lim_{x\to 16}\frac{2\sqrt{x}+x^{3/2}}{\sqrt[4]{x}+5}=\frac{2\sqrt{16}+\left(\sqrt{16}\right)^3}{\sqrt[4]{16}+5}=\frac{2\cdot 4+4^3}{2+5}=\frac{8+64}{7}=\frac{72}{7}$

16. $\lim_{x\to -8}\frac{16x^{2/3}}{4-x^{4/3}}=\frac{16\cdot\left(\sqrt[3]{-8}\right)^2}{4-\left(\sqrt[3]{-8}\right)^4}=\frac{16\cdot(-2)^2}{4-(-2)^4}=\frac{16\cdot 4}{4-16}=\frac{64}{-12}=-\frac{16}{3}$

17. $\lim_{h\to 0}\frac{4-\sqrt{16+h}}{h}=\lim_{h\to 0}\frac{\left(4-\sqrt{16+h}\right)}{h}\cdot\frac{\left(4+\sqrt{16+h}\right)}{\left(4+\sqrt{16+h}\right)}=\lim_{h\to 0}\frac{16-16-h}{h\left(4+\sqrt{16+h}\right)}=$
$\lim_{h\to 0}\frac{-1}{\left(4+\sqrt{16+h}\right)}=\frac{-1}{4+\sqrt{16+0}}=-\frac{1}{8}$

18. $\lim_{h\to 0}\left(\frac{1}{h}\right)\left(\frac{1}{\sqrt{1+h}}-1\right)=\lim_{h\to 0}\left(\frac{1}{h}\right)\left(\frac{1-\sqrt{1+h}}{\sqrt{1+h}}\right)=\lim_{h\to 0}\left(\frac{1}{h}\right)\left(\frac{1-\sqrt{1+h}}{\sqrt{1+h}}\right)\cdot\frac{\left(1+\sqrt{1+h}\right)}{\left(1+\sqrt{1+h}\right)}=$
$\lim_{h\to 0}\frac{1-1-h}{h\sqrt{1+h}\left(1+\sqrt{1+h}\right)}=\lim_{h\to 0}\frac{-1}{\sqrt{1+h}\left(1+\sqrt{1+h}\right)}=\frac{-1}{\sqrt{1+0}\left(1+\sqrt{1+0}\right)}=\frac{-1}{1\cdot 2}=-\frac{1}{2}$

19. $\lim_{x\to 1}\frac{(x-1)^5}{x^5-1}=\lim_{x\to 1}\frac{(x-1)^5}{(x-1)(x^4-x^3+x^2-x+1)}=\lim_{x\to 1}\frac{(x-1)^4}{(x^4-x^3+x^2-x+1)}=\frac{(1-1)^4}{1^4-1^3+1^2-1+1}$
$=\frac{0}{1}=0$ (Must use either long division or synthetic substitution to eliminate offending factor)

20. $\lim_{x\to -1}\frac{(4x^2+5x-3)^3}{(6x+4)^4}=\frac{\left(4(-1)^2+5(-1)-3\right)^3}{\left(6(-1)+4\right)^4}=\frac{\left(4-5-3\right)^3}{\left(-6+4\right)^4}=\frac{(-4)^3}{(-2)^4}=\frac{-64}{16}=-4$

21. $\lim_{x\to 9}\frac{x^2-81}{3-\sqrt{x}}=\lim_{x\to 9}\frac{(x+9)(x-9)}{\left(3-\sqrt{x}\right)}=\lim_{x\to 9}\frac{(x+9)\cdot -1(3-\sqrt{x})(3+\sqrt{x})}{\left(3-\sqrt{x}\right)}=$

$\lim_{x\to 9}\left(-1\right)\cdot(x+9)(3+\sqrt{x})=-1\cdot(9+9)(3+\sqrt{9})=-1(18)(6)=-108$

22. $\lim_{x\to 8}\frac{x-8}{\sqrt[3]{x}-2}=\lim_{x\to 8}\frac{\left(\sqrt[3]{x}-2\right)\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)}{\left(\sqrt[3]{x}-2\right)}=\lim_{x\to 8}\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)=$

$\sqrt[3]{8^2}+2\sqrt[3]{8}+4=4+2\cdot 2+4=12$