Limits Worksheet

1. $\lim_{x\to 2} x^3 - 2x + 1 = (2)^3 - 2(2) + 1 = 8 - 4 + 1 = 5$

2. $\lim_{x\to 0} \frac{x^2 - 2x - 3}{x^2 + 4x + 3} = \lim_{x\to 0} \frac{(x-3)(x+1)}{(x+3)(x+1)} = \lim_{x\to 0} \frac{(x-3)}{(x+3)} = \frac{0-3}{0+3} = \frac{-3}{3} = -1$

3. $\lim_{x\to 1} \frac{x^2 + x}{x^2 + x - 2} = \lim_{x\to 1} \frac{x(x+1)}{(x+2)(x-1)} = \frac{1(1+1)}{(1+2)(1-1)} = \frac{2}{3\cdot 0} = undefined$

4. $\lim_{x\to 1} \frac{x - 1}{x^2 + x - 2} = \lim_{x\to 1} \frac{(x-1)}{(x+2)(x-1)} = \frac{(1-1)}{(1-2)(1-1)} = \frac{0}{0} = undefined$

5. $\lim_{x\to 2} \frac{x^2 - 4}{x^2 + x - 2} = \lim_{x\to 2} \frac{(x+2)(x-2)}{(x+2)(x-1)} = \lim_{x\to 2} \frac{(x-2)}{(x-1)} = \frac{2-2}{2-1} = \frac{0}{1} = 0$

6. $\lim_{x\to 1} \frac{x^2 - 1}{x^2 + x - 2} = \lim_{x\to 1} \frac{(x+1)(x-1)}{(x+2)(x-1)} = \lim_{x\to 1} \frac{(x+1)}{(x+2)} = \frac{1+1}{1+2} = \frac{2}{3}$

7. $\lim_{x\to 1} \frac{x^2 - 1}{x^2 - 2x + 1} = \lim_{x\to 1} \frac{(x+1)(x-1)}{(x-1)(x-1)} = \lim_{x\to 1} \frac{(x+1)}{(x-1)} = \frac{1+1}{1-1} = \frac{2}{0} = undefined$

8. $\lim_{x\to 1} \frac{x^2 - 2x + 1}{x^2 + 3x + 2} = \lim_{x\to 1} \frac{(x-1)(x-1)}{(x+2)(x+1)} = \frac{(1-1)(1-1)}{(1+2)(1+1)} = \frac{0\cdot 0}{3\cdot 2} = 0$

9. $\lim_{x\to 2} \frac{x^2 - 9}{x + 3} = \lim_{x\to 2} \frac{(x+3)(x-3)}{(x+3)} = 2 - 3 = -1$

10. $\lim_{x\to -3} \frac{x^2 - 9}{x + 3} = \lim_{x\to -3} \frac{(x+3)(x-3)}{(x+3)} = -3 - 3 = -6$

11. $\lim_{x\to 0} x^{-1} + 1 + x = \lim_{x\to 0} \frac{1}{x} + 1 + x = \lim_{x\to 0} \frac{1 + x + x^2}{x} = \frac{1+0+0}{0} = undefined$

12. $\lim_{x\to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x\to 4} \frac{(\sqrt{x} - 2)}{(\sqrt{x}+2)(\sqrt{x}-2)} = \lim_{x\to 4} \frac{1}{(\sqrt{x}+2)} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$

13. $\lim_{x\to -2} \frac{x^3 - 2x^2 - 8x}{x^2 + 2x} = \lim_{x\to -2} \frac{x(x-4)(x+2)}{x(x+2)} = -2 - 4 = -6$

14. $\lim_{x\to 3}\frac{x^3-6x+2}{x^2+2x-3}=\frac{(3)^3-6(3)+2}{(3)^2+2(3)-3}=\frac{27-18+2}{9+6-3}=\frac{11}{12}$

15. $\lim_{x\to -5}\frac{(5+x)^2-25}{x}=\frac{(5-5)^2-25}{-5}=\frac{-25}{-5}=5$

16. $\lim_{x\to 1}\frac{x^2-1}{x^3+5x^2-6x}=\lim_{x\to 1}\frac{(x+1)(x-1)}{x(x+6)(x-1)}=\frac{1+1}{1(1+6)}=\frac{2}{7}$

17. $\lim_{x\to 0}\frac{(3+x)^{-1}-3^{-1}}{x}=\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{\frac{3-(3+x)}{3(3+x)}}{x}=\lim_{x\to 0}\frac{-x}{3(3+x)x}=\frac{-1}{3(3+0)}=-\frac{1}{9}$

18. $\lim_{x\to 1}x^2-x+\sqrt{2x-1}=1^2-1+\sqrt{2(1)-1}=1-1+1=1$

19.

$$\lim_{x\to -4}\frac{\sqrt{x^2+9}-5}{x+4}=\lim_{x\to -4}\frac{\left(\sqrt{x^2+9}-5\right)}{\left(x+4\right)}\cdot\frac{\left(\sqrt{x^2+9}+5\right)}{\left(\sqrt{x^2+9}+5\right)}=\lim_{x\to -4}\frac{x^2+9-25}{\left(x+4\right)\left(\sqrt{x^2+9}+5\right)}=$$

$$\lim_{x\to -4}\frac{x^2-16}{\left(x+4\right)\left(\sqrt{x^2+9}+5\right)}=\lim_{x\to -4}\frac{(x+4)(x-4)}{\left(x+4\right)\left(\sqrt{x^2+9}+5\right)}=\frac{-4-4}{\sqrt{(-4)^2+9}+5}=\frac{-8}{10}=\frac{-4}{5}$$

20.

$$\lim_{x\to 0}\left(\frac{1}{x\sqrt{1+x}}-\frac{1}{x}\right)=\lim_{x\to 0}\left(\frac{1-\sqrt{1+x}}{x\sqrt{1+x}}\right)=\lim_{x\to 0}\left(\frac{1-\sqrt{1+x}}{x\sqrt{1+x}}\right)\cdot\frac{\left(1+\sqrt{1+x}\right)}{\left(1+\sqrt{1+x}\right)}=$$

$$\lim_{x\to 0}\frac{1-1+x}{x\sqrt{1+x}\left(1+\sqrt{1+x}\right)}=\lim_{x\to 0}\frac{1}{\sqrt{1+x}\left(1+\sqrt{1+x}\right)}=\frac{1}{\sqrt{1+0}\left(1+\sqrt{1+0}\right)}=\frac{1}{1(2)}=\frac{1}{2}$$

21.

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)}{x}\cdot\frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\lim_{x\to 0}\frac{1+x-1+x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}=$$

$$\lim_{x\to 0}\frac{2}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}=\frac{2}{2}=1$$

22. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}=\frac{\sqrt{1+0}-\sqrt{1-0}}{\sqrt{1+0}+\sqrt{1-0}}=\frac{1-1}{1+1}=\frac{0}{2}=0$

23.
$$\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}=\lim_{x\to 0}\frac{\left(\sqrt{9+x}-3\right)}{x}\cdot\frac{\left(\sqrt{9+x}+3\right)}{\left(\sqrt{9+x}+3\right)}=\lim_{x\to 0}\frac{9+x-9}{x\left(\sqrt{9+x}+3\right)}=$$
$$\lim_{x\to 0}\frac{1}{\left(\sqrt{9+x}+3\right)}=\frac{1}{\sqrt{9+0}+3}=\frac{1}{3+3}=\frac{1}{6}$$

24. $\lim_{x\to 1}\frac{\sqrt{1-x}}{x-1}=\mathit{undefined}$

25.
$$\lim_{x\to 0}\frac{\sqrt{1+2x}-\sqrt{1-3x}}{x}=\lim_{x\to 0}\frac{\left(\sqrt{1+2x}-\sqrt{1-3x}\right)}{x}\cdot\frac{\left(\sqrt{1+2x}+\sqrt{1-3x}\right)}{\left(\sqrt{1+2x}+\sqrt{1-3x}\right)}=$$
$$\lim_{x\to 0}\frac{1+2x-1+3x}{x\left(\sqrt{1+2x}+\sqrt{1-3x}\right)}=\lim_{x\to 0}\frac{5}{\left(\sqrt{1+2x}+\sqrt{1-3x}\right)}=\frac{5}{\sqrt{1+2(0)}+\sqrt{1-3(0)}}=\frac{5}{2}$$

26.
$$\lim_{x\to 0}\frac{x}{\sqrt{x^2+2x}-x}=\lim_{x\to 0}\frac{x}{\left(\sqrt{x^2+2x}-x\right)}\cdot\frac{\left(\sqrt{x^2+2x}+x\right)}{\left(\sqrt{x^2+2x}+x\right)}=\lim_{x\to 0}\frac{x\left(\sqrt{x^2+2x}+x\right)}{x^2+2x-x^2}=$$
$$\lim_{x\to 0}\frac{\left(\sqrt{x^2+2x}+x\right)}{2}=\frac{\sqrt{0+0}+0}{2}=0$$