

Integration

Simplify:

1. $\int 8x dx$

$8x$

2. $\int x^{-5} dx$

$\frac{x^{-4}}{-4}$

3. $\int 2x - 7 dx$

$\frac{2x^2}{2} - 7x$

4. $\int \frac{1}{3x} dx$ $\frac{1}{3} \int \frac{1}{u} du$

$\frac{1}{3} \ln 3x$

5. $\int \sin 5x dx$ $\frac{1}{5} \int \sin u du$

$-\frac{1}{5} \cos 5x$

6. $\int \cos(x+3) dx$

$\int \cos u du$

$\sin(x+3)$

7. $\int e^{-3x} dx$ $-\frac{1}{3} \int e^u du$

$-\frac{1}{3} e^{-3x}$

8. $\int x(x-1) dx$

$\frac{x^3}{3} - \frac{x^2}{2}$

9. $\int \frac{1}{x^2+5^2} dx$

$\frac{1}{5} \arctan \frac{x}{5}$

Substitution:

1. $\int (2x^3+1)^7 x^2 dx$

$u = 2x^3+1$ $\frac{1}{6} \int u^7 du$

$du = 6x^2 dx$

$\frac{1}{6} du = x^2 dx$ $\frac{1}{6} \frac{u^8}{8} = \frac{1}{48} (2x^3+1)^8$

2. $\int x \sin(x^2) dx$

$u = x^2$

$du = 2x dx$

$\frac{1}{2} du = x dx$

$\frac{1}{2} \int \sin u du$

$-\frac{1}{2} \cos x^2$

3. $\int \sin^3 x \cos x dx$

$u = \sin x$

$du = \cos x dx$

$\int u^3 du$

$\frac{u^4}{4} = \frac{(\sin x)^4}{4}$

4. $\int \frac{x^5+x^3+2}{1+x^2} dx = \int x^3 + \frac{2}{1+x^2} dx$

$\frac{x^2+1}{x^5} \frac{x^3}{1+x^2}$

$\frac{x^4}{4} + 2 \arctan x$

20

By Parts:

1. $\int x^2 e^{2x} dx$

$u: x^2 \quad v: \frac{1}{2} e^{2x}$

$du: 2x dx \quad dv: e^{2x} dx$

$\frac{1}{2} \frac{1}{2} e^{2x} x^2 - \int x e^{2x} dx$

$u: x \quad v: \frac{1}{2} e^{2x}$

$du: dx \quad dv: e^{2x} dx$

$\frac{1}{2} e^{2x} x^2 - \left[\frac{1}{2} e^{2x} \cdot x - \frac{1}{2} \int e^{2x} dx \right]$

$\frac{1}{2} e^{2x} x^2 - \frac{1}{2} e^{2x} \cdot x + \frac{1}{4} e^{2x}$

10

2. $\int e^{2x} \cos x dx$

$u: \cos x \quad v: \frac{1}{2} e^{2x}$

$du: -\sin x dx \quad dv: \int e^{2x} dx$

$\frac{1}{2} e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x dx$

$u: \sin x \quad v: \frac{1}{2} e^{2x}$

$du: \cos x \quad dv: \int e^{2x} dx$

$\frac{1}{2} e^{2x} \cos x + \frac{1}{2} \left[\frac{1}{2} e^{2x} \sin x - \frac{1}{2} \int e^{2x} \cos x dx \right]$

$\frac{1}{2} e^{2x} \cos x + \frac{1}{4} e^{2x} \sin x - \frac{1}{4} \int e^{2x} \cos x dx$

$\frac{5}{4} \int e^{2x} \cos x dx = \int e^{2x} \cos x dx$

$\frac{4}{10} e^{2x} \cos x + \frac{4}{20} e^{2x} \sin x$

By Partial fractions:

1. $\int \frac{5x^2 - 10x - 8}{x^3 - 4x} dx$

$x(x-2)(x+2)$

$\int \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x+2)}$

$A(x^2-4) + B(x)(x+2) + C(x)(x-2)$

$Ax^2 - 4A + Bx^2 + 2Bx + Cx^2 - 2Cx$

$A+B+C = 5 \quad B+C = 3$

$2B - 2C = -10 \quad B - C = -5$

$-4A = -8 \quad 2B = -2$

$A = 2 \quad B = -1$

$C = 4$

$\int \frac{2}{x} dx + \int \frac{-1}{(x-2)} dx + \int \frac{4}{(x+2)} dx$

$2 \ln|x| - 1 \ln|x-2| + 4 \ln|x+2|$

15

2. $\int \frac{1}{x^2(x-1)(x^2+3)(x^3+2x^2-x+1)} dx$: write in the form of partial fractions but do not solve.

2 $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{Dx+E}{x^2+3} + \frac{Fx^2+Gx+H}{x^3+2x^2-x+1}$

Definite Integral:

1. Evaluate: $\int_1^5 (3x^2 + 4x + 1) dx$

4 $\frac{3x^3}{3} + \frac{4x^2}{2} + x \Big|_1^5 = \left[\frac{3(5)^3}{3} + \frac{4(5)^2}{2} + 5 \right] - \left[\frac{3(1)^3}{3} + \frac{4(1)^2}{2} + 1 \right]$
 $[125 + 50 + 5] - [1 + 2 + 1]$
 $180 - 4$
 176

2. Find the displacement of an object from $t = 2$ to $t = 3$, if the velocity of the object at time t is given by:

4 $\int_2^3 \frac{t^2 + 1}{t^3 + 3t} dt$ $\frac{1}{3} \int_2^3 \frac{1}{u^2} du = \frac{1}{3} \left[\frac{u^{-1}}{-1} \right]_2^3$
 $u = t^3 + 3t$
 $du = 3t^2 + 3 dt$
 $du = 3(t^2 + 1) dt$
 $\frac{1}{3} du = (t^2 + 1) dt$
 $\left[\frac{1}{3(t^3 + 3t)} \right]_2^3 = \frac{1}{3(27 + 9)} - \frac{1}{3(8 + 6)}$
 $\left[\frac{1}{3(36)} - \frac{1}{3(14)} \right]$
 $\frac{1}{108} - \frac{1}{42} = \frac{42}{42 \cdot 108} - \frac{108}{42 \cdot 108} = \frac{-66}{42 \cdot 108} = \frac{-33}{21 \cdot 108}$

10