Some Properties Of The Real Numbers

A real number is either a positive number, a negative number, or zero and may be classified as a:

a) ***rational*** - a number that can be expressed as a ratio of two integers. These include integers, positive and negative fractions, terminating decimals, and non-terminating repeating decimals. Examples: -5, 3, 2/7, -4/5, 2.45, -0.00561, 0.6666..., -3.456456...

b) ***irrational*** - any number that is not rational. Examples:

$$\sqrt{17} = 4.1231...., \quad \pi = 3.14159..., \quad \tan\frac{8\pi}{9} = -0.364...$$

Fundamental Properties of the Operations of Addition and Multiplication.
{a, b, and c are real numbers}

1. Commutative Laws:	a + b = b + a	ab = ba
2. Associative Laws	a + (b + c) = (a + b) + c	a(bc) = (ac)b
3. Distributive Law	a(b + c) = ab + ac	
4. Identity Elements	a + 0 = a	a x 1 = a
5. Existence of Negatives	a + (-a) = 0	
6. Existence of Reciprocals	every real number $a \neq 0$ has a reciprocal, denoted by 1/a, such that *a x 1/a = 1*	
7. Zero Product Property	*ab = 0* if and only if *a= 0* or *b = 0*	

Inequalities

Definition:

a) The symbols $<$ ("is less than") and $>$ ("is greater than") are defined as follows:

1. $a < b$ if and only if $b - a$ is positive
2. $a > b$ if and only if $a - b$ is positive

b) The symbols $\leq$ ("is less than or equal to") and $\geq$ ("is greater than or equal to") are defined as follows:

1. $a \leq b$ if and only if either $a < b$ or $a = b$
2. $a \geq b$ if and only if either $a > b$ or $a = b$

Properties:

1. If $a < b$ and $b < c$, then $a < c$
2. If $a < b$, then $a + c < b + c$, if "c" is any real number
3. If $a < b$ and $c < d$, then $a + c < b + d$
4. If $a < b$, and "c" is any positive number, then $ac < bc$
5. If $a < b$, and "c" is any negative number, then $ac > bc$
6. If $a > b$ and $b > c$, then $a > c$
7. If $a > b$, then $a + c > b + c$, if "c" is any real number
8. If $a > b$ and $c > d$, then $a + c > b + d$
9. If $a > b$, and "c" is any positive number, then $ac > bc$
10. If $a > b$, and "c" is any negative number, then $ac < bc$

Intervals

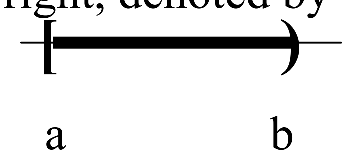
Definitions:

a) The bounded open interval from a to b, denoted by (a, b), is the set of real numbers x such that a < x < b.

b) The bounded closed interval from a to b, denoted by [a, b], is the set of real numbers x such that $a \leq x \leq b$

c) The bounded interval half-open on the left, denoted by (a, b], is the set of all real numbers x such that $a < x \leq b$

d) The bounded interval half-open on the right, denoted by [a, b), is the set of all real numbers x such that $a \leq x < b$



Infinity

Symbols: $\infty \rightarrow$ infinity

$+\infty$ *or* $\infty \rightarrow$ positive inf

$-\infty \rightarrow$ negative infinity

$(-\infty, +\infty)$ or $(-\infty, \infty)$

denotes the set of all real numbers

Unbounded Open Interval

$(a, +\infty)$ denotes: a) the set of all numbers greater than a

b) the set of all real numbers x such that $x > a$

Unbounded Closed Interval

$[a, +\infty)$ denotes a) the set of all numbers greater than or equal to a

b) the set of all real numbers x such that $x \geq a$

Unbounded Open Interval

$(-\infty, b)$ denotes: a) the set of all numbers less than b

b) the set of all real numbers x such that $x < b$

Unbounded Closed Interval

$(-\infty, b]$ denotes: a) the set of all numbers less than or equal to b

b) the set of all real numbers x such that $x \leq b$

Absolute Value

If a is a real number, then the absolute value of a is:

$$|a| = \begin{cases} a, & if \;\; a \geq 0 \\ -a, & if \;\; a < 0 \end{cases}$$

The absolute value of a number cannot be negative. The symbol -a does not necessarily mean that -a is negative.

Operations with absolute values: (*let a and b be real numbers and n a positive integer*)

$$1.\;\; |ab| = |a|\,|b| \qquad 2.\;\; \left|\frac{a}{b}\right| = \frac{|a|}{|b|},\;\; b \neq 0$$

$$3.\;\; |a| = \sqrt{a^2} \qquad 4.\;\; |a^n| = |a|^n$$

Properties of inequalities and absolute values
(*let a and b be real numbers and n a positive integer*)

$$1.\;\; -|a| \leq a \leq |a|$$

$$2.\;\; |a| \leq k \;\; if\; and\;\; only\;\; if \;\; -k \leq a \leq k$$

$$3.\;\; k \leq |a| \;\; if\; and\;\; only\;\; if \;\; k \leq a \;\; or \;\; a \leq -k$$

$$4.\;\; Triangle \;\; Inequality \;\; |a+b| \leq |a| + |b|$$

Properties 2 and 3 are also true if $\leq$ is replaced by $<$

Distance and Midpoint of an Interval

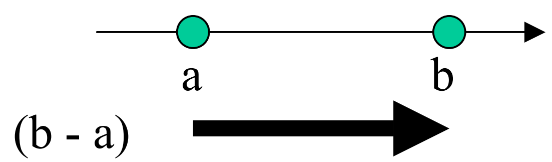
Distance:

The distance between two points a and b on the real line is given by:

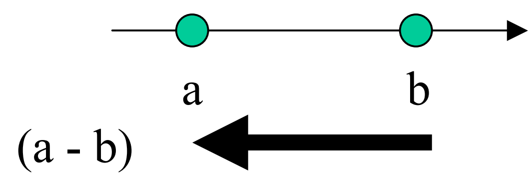
$$d = |a - b| = |b - a|$$

The directed distance from a to b is (b - a) and the directed distance from b to a is (a - b)

directed distance
from a to b



directed distance
from b to a



Midpoint

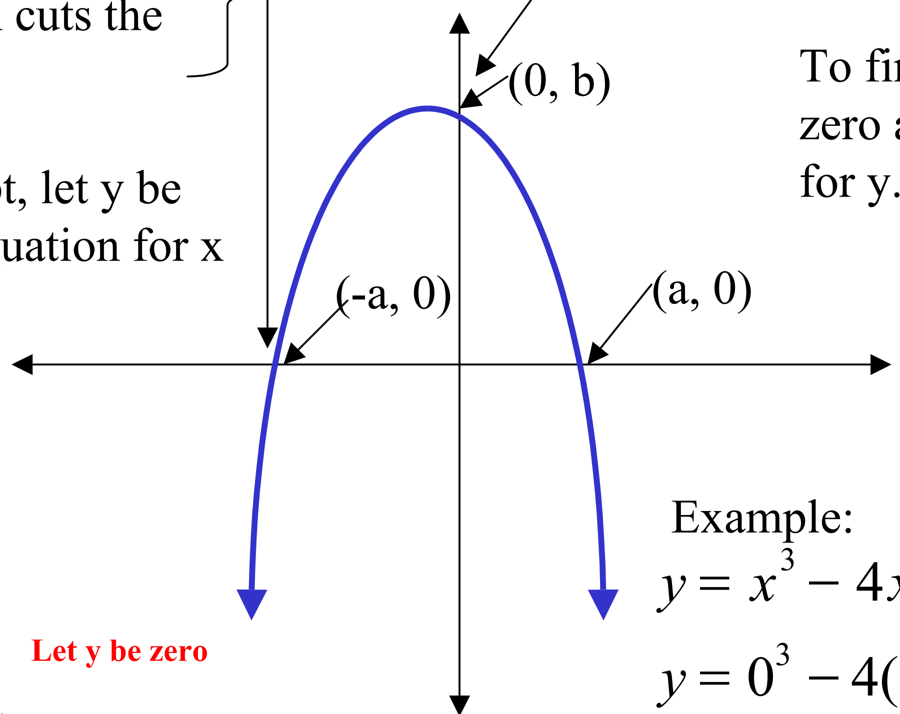
The midpoint of an interval with endpoints a and b is the average of a and b.

$$M = \frac{a + b}{2}$$

Intercepts of a Graph

X-intercept - the value of x at the point where the graph cuts the x-axis.

Y-intercept - the value of y at the point where the graph cuts the y-axis.



To find the x-intercept, let y be zero and solve the equation for x

To find a y-intercept, let x be zero and solve the equation for y.

Example:

$y = x^3 - 4x$

$0 = x^3 - 4x$ **Let y be zero**

$0 = x(x - 2)(x + 2)$ **Factor**

$x = 0, 2, -2$ **Solve for x**

Example:

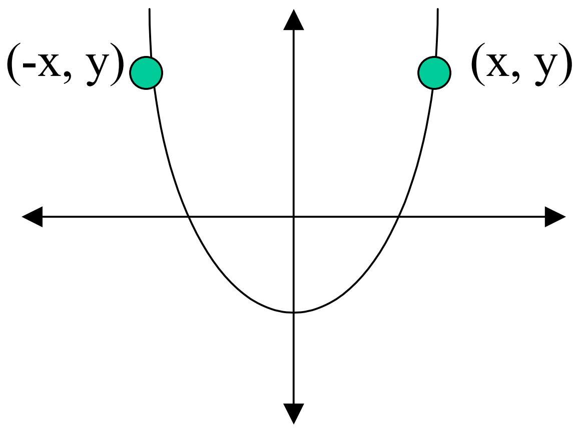
$y = x^3 - 4x$

$y = 0^3 - 4(0)$ **Let x be zero**

$y = 0$ **Solve for y**

Symmetry of a Graph

1. A graph is symmetric with respect to the y-axis, if whenever (x, y) is a point on the graph, (-x, y) is also a point on the graph. This means that the portion of the graph to the left of the y-axis is a mirror image of the portion to the right of the y-axis.



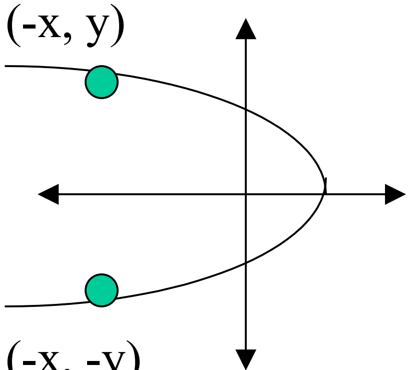
Test

The graph of an equation in x and y is symmetric with respect to the y-axis if replacing x by -x yields equivalent equation.

Example:

$y = 2x^2 - 3$	original equation
$y = 2(-x)^2 - 3$	replace x by - x
$y = 2x^2 - 3$	simplify
$y = 2x^2 - 3$	equivalent equatio[n]

2. A graph is symmetric with respect to the x-axis, if whenever (x, y) is a point on the graph, (x, -y) is also a point on the graph. This means that the portion of the graph above the x-axis is a mirror image of the portion below the x-axis.

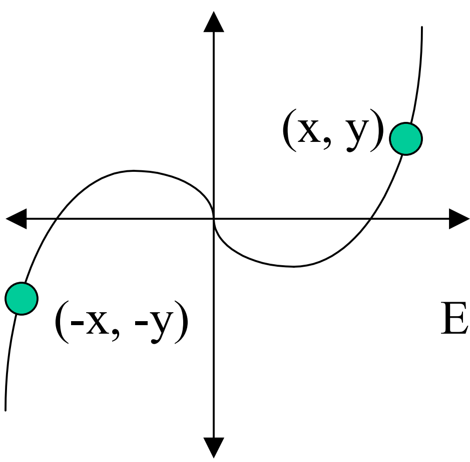


Test: The graph of an equation in x and y is symmetric with respect to the x-axis if replacing y by -y yields An equivalent equation.

Example

$x = -2y^2 + 3$	original equation
$x = -2(-y)^2 + 3$	replace y by - y
$x = -2y^2 + 3$	simplify
$x = -2y^2 + 3$	equivalent equation

3. A graph is symmetric with respect to the origin, if whenever (x, y) is a point on the graph, (-x, -y) is also a point on the graph. This means that the graph is unchanged by a rotation of 180 degrees about the origin.



Test: The graph of an equation in x and y is symmetric with respect to the origin if replacing x by -x and y by -y yields an equivalent equation.

Example

$y = 3x^3 - 2x$	original equation
$-y = 3(-x)^3 - 2(-x)$	replace x by - x and y by - y
$-y = -3x^3 + 2x$	simplify
$y = 3x^3 - 2x$	equivalent equation

Basic Formulas

Pythagorean Theorem - states that in a right triangle the hypotenuse c and sides a and b are related by $c^2 = a^2 + b^2$ and conversely, if $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Distance Formula - the distance between the points (x_1, x_2) *and* (y_1, y_2) in the plane is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula - the midpoint is found by averaging the x-coordinates of the two points and averaging the y-coordinates of the two points $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example: Find 'x' so that the distance between (x, 3) and (2, -1) is 5 and then calculate the midpoint of the line segment joining the two points. Determined points (5, 3) or (-1, 3)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	distance formula
$5 = \sqrt{(x-2)^2 + [3-(-1)]^2}$	substitution
$25 = (x^2 - 4x + 4) + 16$	square both sides
$0 = x^2 - 4x - 5$	
$0 = (x-5)(x+1)$	factor
$x = 5 \ \ or \ \ -1$	solve

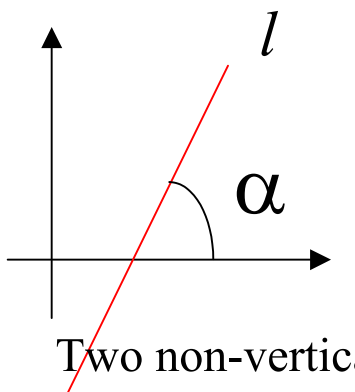
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	formula
$M\left(\frac{5+2}{2}, \frac{3+(-1)}{2}\right)$	substitution
$M\left(\frac{7}{2}, \frac{2}{2}\right) = \left(\frac{7}{2}, 1\right)$	simplify

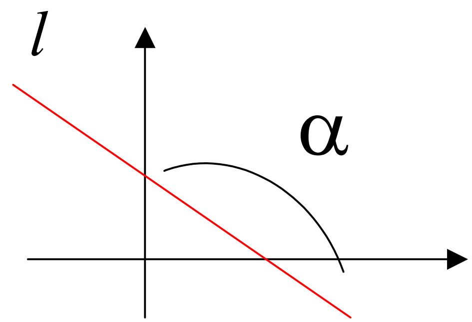
Slope: if $P_1(x_1, x_2)$ *and* $P_2(x_2, y_2)$ are any two distinct points on the line *l* , which is not parallel to the y-axis, then the slope of *l* , denoted by *m* , is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta x}{\Delta y} \quad \text{where} \quad \Delta \text{ (delta) means " a change in"}$$

The value of *m* computed from the formula is independent of the choice of the two points on on the given line.

The inclination of a line not parallel to the x-axis is the smallest angle measured counterclockwise from the positive direction of the x-axis to the line. The inclination of a line parallel to the x-axis is defined to be zero. If α denotes the inclination of a line, α may be any angle in the interval $0 \leq \alpha < \pi$





If α is the inclination of line *l,* not parallel to the y-axis, then the slope *m* of *l* is given by:

$$m = \tan \alpha$$

Two non-vertical lines are parallel if and only if their slopes are equal $m_1 = m_2$ and they are perpendicular if and only if the product of their slopes is equal to -1. $m_1 \cdot m_2 = -1$he slopes are considered negative reciprocals of one another.