Calculus 30- Typical Exam Questions

A. Determine the domain and the range of the following:

1. $y = -x^2 + 4$

   D: $(-\infty, \infty)$

   R: $(-\infty, 4]$

2. $y = \sqrt{16 - x^2}$

   D: $[-4, 4]$

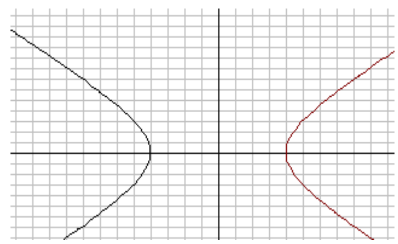
   R: $[0, 4]$

3. $y = \dfrac{1}{(x+6)(x-1)}$

   D: $(-\infty, -6) \cup (-6, 1) \cup (1, \infty)$

   R: $(-\infty, 0) \cup (0, \infty)$

4.



   D: $(-\infty, -4] \cup [4, \infty)$

   R: $(-\infty, \infty)$

B. Given the domain and range, determine the equation of the following:

1. domain: $(-\infty, -4] \cup [4, \infty)$, range $[0, \infty)$ $\quad y = \sqrt{x^2 - 16}$

2. domain: $[-6, 6]$, range [0, 6] $\quad y = \sqrt{36 - x^2}$

3. domain: $(-\infty, -4) \cup (-4, 5) \cup (5, \infty)$, range $(-\infty, 0) \cup (0, \infty)$ $\quad y = \dfrac{1}{(x+4)(x-5)}$

C. Given: $f(x) = -5x - 2$ and $g(x) = -2x^2 + 1$, determine:

1. f(x) + f(x)

   $(-5x - 2) + (-5x - 2)$

   $-10x - 4$

2. f(x) * g(x)

   $(-5x - 2)(-2x^2 + 1)$

   $10x^3 - 5x + 4x^2 - 2$

3. $(g \circ f)x$

   $-2(-5x - 2)^2 + 1$

   $-2(25x^2 + 20x + 4) + 1$

   $-50^2 - 40x - 8 + 1$

   $-50x^2 - 40x - 7$

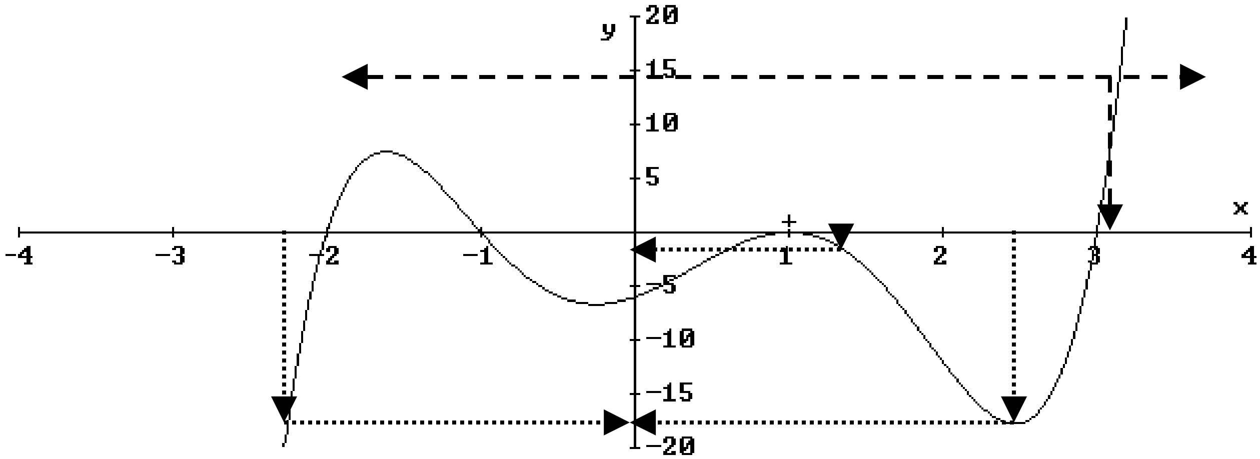
4. $(f \circ g)x$

   $-5(-2x^2 + 1) - 2$

   $10x^2 - 5 - 2$

   $10x^2 - 7$

D. Determine the value of f(x) from the given graph:



Note: Lines are in approximate location

f(-2.2) = -17 f(2.5) = -18 f(1.25) = -1 f(3.2) = 15

1. $\lim_{x \to 4} x^3 - 5x + 2$

$(4)^3 - 5(4) + 2$

46

2. $\lim_{x \to -2} \frac{6x^2 - 5}{x - 2}$

$\frac{6(-2)^2 - 5}{(-2) - 2}$

$\frac{-19}{4}$

3. $\lim_{x \to 3} \frac{x^2 - 5x + 6}{x - 3}$

$\frac{(x-3)(x-2)}{(x-3)}$

$3 - 2$

1

4. $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$

$\frac{(\sqrt{x+4} - 2)}{x} \cdot \frac{(\sqrt{x+4} + 2)}{(\sqrt{x+4} + 2)}$

$\frac{x + 4 - 4}{x(\sqrt{x+4} + 2)}$

$\frac{1}{(\sqrt{0+4} + 2)}$

$\frac{1}{4}$

5.

$$\lim_{x \to 0} \frac{\sin 4x}{5x}$$

$$\frac{\frac{4}{5}\sin(4x)}{\frac{4}{5}(5x)}$$

$$\frac{4}{5} \cdot \frac{\sin(4x)}{4x}$$

$$\frac{4}{5}$$

6.

$$\lim_{x \to 0} \frac{\sin x - \sin x \cos x}{x^2}$$

$$\frac{\sin x(1-\cos x)}{x \cdot x}$$

$$\frac{\sin x}{x} \cdot \frac{1-\cos x}{x}$$

$$1 \cdot 0$$

$$0$$

7.

$$\lim_{x \to 25} \frac{x-25}{\sqrt{x}-5}$$

$$\frac{(\sqrt{x}+5)(\sqrt{x}-5)}{(\sqrt{x}-5)}$$

$$\sqrt{25}+5$$

$$10$$

8.

$$\lim_{x \to \infty} \frac{5x^3 - x}{x^2+3}$$

$$\infty$$

9.

$$\lim_{x \to \infty} \frac{7x^3 - 5x^2 + 6}{4x^3 + 2x - 7}$$

$$\frac{7}{4}$$

10.

$$\lim_{x \to \infty} \frac{3x^2 - 5x - 2}{4x^5 + 3x - 9}$$

$$0$$

11.

$$\lim_{x \to 2} \frac{x^3 - 8}{x-2}$$

$$\frac{(x-2)(x^2+2x+4)}{(x-2)}$$

$$(2)^2 + 2(2) + 4$$

$$12$$

12,

$$\lim_{x \to 4^+} \frac{5}{4-x}$$

$$\frac{5}{4-(4.00001)}$$

$$\frac{5}{-.00001}$$

$$-500000 \to -\infty$$

13. $\lim_{x \to 2^-} \frac{1}{x^2 - 5x + 6}$

$\frac{1}{(1.999)^2 - 5(1.999) + 6}$

$\frac{1}{.001}$

$1000 \to \infty$

14. $\lim_{x \to -\infty} x^3 + 2x^2$

$(-\infty)^3 + 2(-\infty)^2$

$(-\infty)^3 + \infty^2$

$-\infty$

15. Determine the equation of the tangent line to the curve $f(x) = x^4 + x + 1$ at x = -2. (do not use derivatives)

$f(x) = x^4 + x + 1$

$y = (-2)^4 + (-2) + 1$ therefore the point is (-2, 15)

$y = 16 - 2 + 1$

$y = 15$

$f(x) = x^4 + x + 1$

a second point $y = (-2.0001)^4 + (-2.0001) + 1$ (-2.0001, 15.003)

$y = 15.003$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 15.003}{-2 - (-2.0001)} = \frac{-.003}{.0001} = 30$$

(y - 15) = 30(x - (-2))
y - 15 = 30x + 60
y = 15x + 75