

Optimization Problems

1. Find the maximum and minimum values of $f(x) = x^3 - 3x$ on the interval $[-2, 2]$.
{-2, -1, 1, 2}
2. Find the maximum and minimum values of $f(x) = 2 + 2x - 3x^{\frac{2}{3}}$ on the interval $[-1, 2]$. {-1, 0}
3. Find two nonnegative numbers whose sum is 10 and the sum of whose squares is a minimum. {5}
4. Find two positive numbers whose sum is 20 and such that its product is as large as possible. {10}
5. Find the rectangle with area 64 inches square for which the perimeter is a minimum. {8}
6. Find the point on the graph of $y = \sqrt{x}$ nearest the point $(4, 0)$. $\left\{\frac{7}{2}, \sqrt{\frac{7}{2}}\right\}$
7. A rectangular field is to be fenced on three sides with 1000m of fencing (the fourth side being a straight river's edge). Find the dimensions of the field in order that the area be as large as possible. {250, 500}
8. An open box is to be made from a square piece of cardboard measuring 12 inches on a side by cutting a square from each corner and folding up the sides. Find the dimensions for which the volume of the resulting box is a maximum. {2, 8, 8}
9. Find the right circular cylinder of maximum volume that can be inscribed in a sphere of radius 10 cm. $\left\{\frac{10\sqrt{6}}{3}, \frac{20\sqrt{3}}{3}\right\}$
10. Pop cans to hold 300ml are made in the shape of right-circular cylinders. Find the dimensions of the can that minimize its surface area. $\left\{\sqrt[3]{150/\pi}, 2\sqrt[3]{150/\pi}\right\}$
11. A length of wire L is to be cut into two pieces, one of which is bent to form a circle and the other to form a square. How should the wire be cut if the sum of the areas enclosed by the two pieces is a maximum? $\left\{\frac{L}{2\pi + 8}, \frac{L}{\pi + 4}\right\}$
12. A square sheet of tin "a" inches on a side is used to make an open-top box by cutting a small square of time from each corner and bending up the sides. How large a square should be cut from each corner in order that the box have as large a volume as possible? {a/6}
13. An apple orchard now has 30 trees per acre, and the average yield is 400 apples per tree. For each additional tree planted per acre, the average yield per tree is reduced by approximately 10 apples. How many trees per acre will give the largest crop of apples? {5}

14. Find the radius and height of the right-circular cylinder of largest volume that can be inscribed in a right circular cone with a radius of 6 inches and a height of 10 inches.
{4, 10/3}
15. A liquid form of penicillin manufactured by a pharmaceutical firm is sold in bulk at a price of \$200 per unit. If the production cost (in dollars) for "x" units is $c(x) = 500,000 + 80x + .003x^2$ and if the production capacity for the firm is at most 30,000 units in a specified time, how many units of penicillin must be manufactured and sold in that time to maximize profit? {20,000}