DERIVATIVES - QUOTIENT RULE WORKSHEET #6

Rule: if $f(x)=\dfrac{g(x)}{h(x)}$ then $f'(x)=\dfrac{g'(x)h(x)-h'(x)g(x)}{[h(x)]^2}$

1.
$$f(x)=\frac{3x^2-x+2}{4x^2+5}$$
$$f'(x)=\frac{(6x-1)(4x^2+5)-(8x)(6x-1)}{\left(4x^2+5\right)^2}$$
$$f'(x)=\frac{24x^3+30x-4x^2-5-48x^2+8x}{\left(4x^2+5\right)^2}$$
$$f'(x)=\frac{24x^3-52x^2+38x-5}{\left(4x^2+5\right)^2}$$

2.
$$f(x)=\frac{(4x-5)}{(3x+2)}$$
$$f'(x)=\frac{(4)(3x+2)-(3)(4x-5)}{[(3x+2)]^2}$$
$$f'(x)=\frac{12x+8-12x+15}{(3x+2)^2}$$
$$f'(x)=\frac{23}{(3x+2)^2}$$

3.
$$f(x)=\frac{(8-x+3x^2)}{(2-9x)}$$
$$f'(x)=\frac{(-1+6x)(2-9x)-(-9)(8-x+3x^2)}{[(2-9x)]^2}$$
$$f'(x)=\frac{-2+9x+12x-54x^2-(-72+9x-27x^2)}{[(2-9x)]^2}$$
$$f'(x)=\frac{-2+9x+12x-54x^2+72-9x+27x^2}{[(2-9x)]^2}$$
$$f'(x)=\frac{-27x^2+12x+70}{[(2-9x)]^2}$$

4.
$$f(x)=\frac{2x}{(x^3-7)}$$
$$f'(x)=\frac{(2)(x^3-7)-(3x^2)(2x)}{\left[(x^3-7)\right]^2}$$
$$f'(x)=\frac{2x^3-14-6x^3}{\left[(x^3-7)\right]^2}$$
$$f'(x)=\frac{-4x^3-14}{\left[(x^3-7)\right]^2}$$

5.
$$f(x)=\frac{(8x^2-5x)}{(13x^2+4)}$$
$$f'(x)=\frac{(16x-5)(13x^2+4)-(26x)(8x^2-5x)}{\left[(13x^2+4)\right]^2}$$
$$f'(x)=\frac{208x^3+64x-65x^2-20-208x^3+130x^2}{\left[(13x^2+4)\right]^2}$$
$$f'(x)=\frac{65x^2+64x-20}{\left[(13x^2+4)\right]^2}$$

6.
$$f(x)=\frac{\left(x^3-1\right)}{\left(x^3+1\right)}$$
$$f'(x)=\frac{3x^2\left(x^3+1\right)-3x^2\left(x^3-1\right)}{\left[\left(x^3+1\right)\right]^2}$$
$$f'(x)=\frac{3x^5+3x^2-3x^5+3x^2}{\left[\left(x^3+1\right)\right]^2}$$
$$f'(x)=\frac{6x^2}{\left[\left(x^3+1\right)\right]^2}$$

$$f(x)=\frac{(8x+5)}{(x^2-2x+3)}$$

$$f'(x)=\frac{(8)(x^2-2x+3)-(2x-2)(8x+5)}{[(x^2-2x+3)]^2}$$

7. $$f'(x)=\frac{8x^2-16x+24-(16x^2+10x-16x-10)}{[(x^2-2x+3)]^2}$$

$$f'(x)=\frac{8x^2-16x+24-16x^2-10x+16x+10}{[(x^2-2x+3)]^2}$$

$$f'(x)=\frac{-8x^2-10x+34}{[(x^2-2x+3)]^2}$$

$$f(x)=\frac{\frac{3}{5x}-1}{\frac{2}{x^2+7}}=\frac{\frac{3-5x}{5x}}{\frac{2}{x^2+7}}=\frac{(3-5x)(x^2+7)}{2\cdot 5x}$$

$$f(x)=\frac{-5x^3+3x^2-35x+21}{10x}$$

8. $$f'(x)=\frac{(-15x^2+6x-35)\cdot 10x-10(-5x^3+3x^2-35x+21)}{[10x]^2}$$

$$f'(x)=\frac{-150x^3+60x^2-350x+50x^3-30x^2+350x+210}{[10x]^2}$$

$$f'(x)=\frac{-100x^3+60x^3-30x^2+210}{[10x]^2}$$

$$f(x)=\frac{e^{-x^2}}{x}$$

9. $$f'(x)=\frac{e^{-x^2}\cdot -2x\cdot(x)-(1)e^{-x^2}}{[x]^2}$$

$$f'(x)=\frac{e^{-x^2}(-2x^2-1)}{[x]^2}$$

$$f(x)=\frac{e^{3x}}{1+e^x}$$

$$f'(x)=\frac{e^{3x}\cdot 3\cdot(1+e^x)-e^x\cdot e^{3x}}{[1+e^x]^2}$$

10. $$f'(x)=\frac{e^{3x}(3+3e^x-e^x)}{[1+e^x]^2}$$

$$f'(x)=\frac{e^{3x}(3+2e^x)}{[1+e^x]^2}$$

11.

$$f(x)=\frac{\ln x}{1+x^2}$$

$$f'(x)=\frac{\frac{1}{x}\left(1+x^2\right)-2x\ln x}{\left[1+x^2\right]^2}$$

$$f'(x)=\frac{\frac{1}{x}+\frac{x^2}{x}-2x\ln x}{\left[1+x^2\right]^2}=\frac{\frac{1}{x}+\frac{x^2}{x}-\frac{2x^2\ln x}{x}}{\left[1+x^2\right]^2}$$

$$f'(x)=\frac{\frac{1+x^2-2x^2\ln}{x}}{\left[1+x^2\right]^2}=\frac{1+x^2-2x^2\ln}{x\left[1+x^2\right]^2}$$

12.

$$f(x)=\frac{1-\ln x}{1+\ln x}$$

$$f'(x)=\frac{-\frac{1}{x}(1+\ln x)-\frac{1}{x}(1-\ln x)}{[1+\ln x]^2}$$

$$f'(x)=\frac{-\frac{1}{x}-\frac{\ln x}{x}-\frac{1}{x}+\frac{\ln x}{x}}{[1+\ln x]^2}$$

$$f'(x)=\frac{-\frac{2}{x}}{[1+\ln x]^2}=-\frac{2}{x[1+\ln x]^2}$$

13.

$$f(x)=\frac{(1+\ln x)^4}{x}$$

$$f'(x)=\frac{4(1+\ln x)^3\cdot\frac{1}{x}\cdot x-1\cdot(1+\ln x)^4}{[x]^2}$$

$$f'(x)=\frac{4(1+\ln x)^3-(1+\ln x)^4}{[x]^2}$$

$$f'(x)=\frac{(1+\ln x)^3\left[4-(1+\ln x)\right]}{[x]^2}$$

$$f'(x)=\frac{(1+\ln x)^3[3-\ln x]}{[x]^2}$$

14.

$$f(x)=\frac{5^x}{2^x}$$

$$f'(x)=\frac{5^x\cdot\ln 5\cdot 2^x-2^x\cdot\ln 2\cdot 5^x}{\left[2^x\right]^2}$$

$$f'(x)=\frac{5^x\cdot 2^x(\ln 5-\ln 2)}{\left[2^x\right]^2}$$

$$f'(x)=\frac{5^x(\ln 5-\ln 2)}{2^x}$$

15.

$$f(x)=\frac{\log_6(2x)}{\ln x}=\frac{\frac{\ln(2x)}{\ln 6}}{\ln x}=\frac{1}{\ln 6}\cdot\frac{\ln(2x)}{\ln(x)}$$

$$f'(x)=\frac{1}{\ln 6}\cdot\left[\frac{\frac{1}{2x}\cdot 2\cdot\ln x-\frac{1}{x}\ln(2x)}{[\ln(x)]^2}\right]\Rightarrow f'(x)=\frac{1}{\ln 6}\cdot\left[\frac{\frac{1}{x}\cdot\ln x-\frac{1}{x}\ln(2x)}{[\ln(x)]^2}\right]$$

$$f'(x)=\frac{1}{\ln 6}\cdot\left[\frac{\frac{1}{x}(\ln x-\ln(2x))}{[\ln(x)]^2}\right]=\frac{1}{\ln 6}\cdot\frac{1}{x}\left[\frac{(\ln x-\ln(2x)}{[\ln(x)]^2}\right]$$

$$f(x)=\frac{3^{(5x-1)}}{e^{4x}}$$

16. $$f'(x)=\frac{3^{(5x-1)}\cdot\ln 3\cdot 5\cdot e^{4x}-e^{4x}\cdot 4\cdot 3^{(5x-1)}}{\left[e^{4x}\right]^2}\Rightarrow f'(x)=\frac{3^{(5x-1)}\cdot e^{4x}\cdot\left(\ln 3\cdot 5-4\right)}{\left[e^{4x}\right]^2}$$

$$f'(x)=\frac{3^{(5x-1)}\cdot\left(\ln 3\cdot 5-4\right)}{e^{4x}}$$