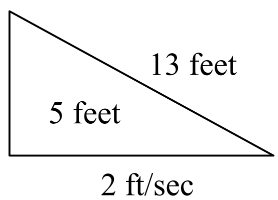
1. Radius of the circle at identified area: $A = \pi r^2 \Rightarrow 9km^2 = \pi r^2 \Rightarrow r = {}^{3km}\!/_{\pi}$

   Givens: $\frac{dA}{dt} = 6km^2hr, \frac{dr}{dt} = ?, r = \frac{3}{\sqrt{\pi}}km$

   $$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt} \Rightarrow 6km^2hr = 2\pi \cdot \frac{3km}{\sqrt{\pi}}\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\sqrt{\pi}}km/hr$$

2. Givens: $\frac{dV}{dt} = ?, r = 9cm, \frac{dr}{dt} = 15cm/\min$

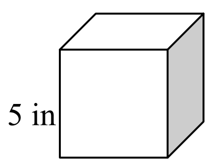
   $$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\frac{dr}{dt} \Rightarrow \frac{dV}{dt} = 4\pi(9cm)^2(15cm/\min) = 4860\pi\, cm^3/\min$$

3. 

   $c^2 = a^2 + b^2 \Rightarrow 13^2 = 5^2 + b^2 \Rightarrow b = 12$

   Givens: $c = 13ft, a = 5ft, b = 12ft, \frac{dc}{dt} = 0, \frac{da}{dt} = -2ft/\sec, \frac{db}{dt} = ?$

   $$c^2 = a^2 + b^2 \Rightarrow 2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} \Rightarrow 13ft(0) = 5ft(-2ft/\sec) + 12ft\frac{db}{dt} \Rightarrow \frac{db}{dt} = \frac{5}{6}ft/\sec$$

4. 

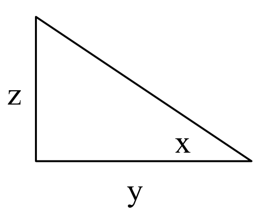
   Determine rate at which side is increasing:

   Givens: x $= 5in, \frac{dv}{dt} = 2in^3/\min, \frac{dr}{dt} = ?$

   $$V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 2in^3/\min = 3(5in)^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = {}^{2}\!/_{75}\, in/\min$$

   Determine the rate of increase in surface area

   $$SA = 6x^2 \Rightarrow \frac{dSA}{dt} = 12x\frac{dx}{dt} \Rightarrow \frac{dSA}{dt} = 12(5in)\cdot\frac{2}{75}in/\min = \frac{8}{5}in^2/\min$$

5. 

   $\tan x = \frac{opp}{adj} \Rightarrow \tan x = \frac{z}{y} = zy^{-1}$

   To determine the size of the angle: $\tan x = \frac{3000}{26400} \Rightarrow x = 6.48^o$

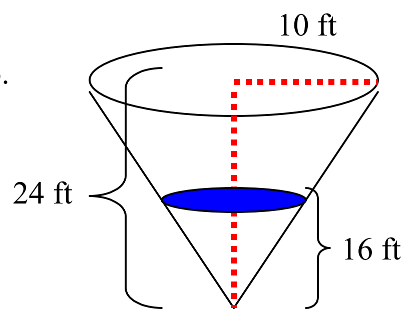
   Givens: $y = 26,400ft, z = 3000ft, x = 6.48^o, \frac{dz}{dt} = 500ft/\sec, \frac{dy}{dt} = 0, \frac{dx}{dt} = ?$

$$\tan x = zy^{-1} \Rightarrow \sec^2 x\frac{dx}{dt} = \frac{dz}{dt}\cdot y^{-1} + -1y^{-2}\frac{dy}{dt}\cdot z$$

$$\left(\sec 6.48^o\right)^2\frac{dx}{dt} = (500\,ft/\sec)\cdot(26,400\,ft)^{-1} - \left(26,400\,ft\right)^{-2}(0)\cdot(3000\,ft)$$

$$1.013\frac{dx}{dt} = \frac{500\,ft/\sec}{26,400\,ft} \Rightarrow 0.187\,rad/\sec = 1.04^o/\sec$$

6.



Ratio of radius to height $\frac{10}{24} = \frac{r}{h} \Rightarrow r = \frac{5}{12}h$

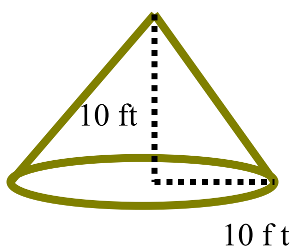
Volume of a Cone:

$$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h \Rightarrow V = \frac{25\pi}{432}h^3$$

Givens: $h = 16\,feet, \frac{dV}{dt} = 20\,ft^3/\min, \frac{dh}{dt} = ?$

$$V = \frac{25\pi}{432}h^3 \Rightarrow \frac{dV}{dt} = \frac{25\pi}{432}\cdot 3h^2\frac{dh}{dt} \Rightarrow 20\,ft^3\,\min = \frac{25\pi}{432}\cdot 3(16\,ft)^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{9}{20\pi}\,ft/\sec$$

7.



Ratio of radius to height $\frac{5}{10} = \frac{r}{h} \Rightarrow r = \frac{h}{2}$

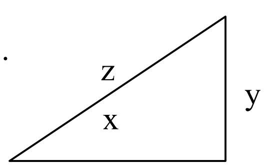
Volume of a Cone:

$$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h \Rightarrow V = \frac{1}{12}\pi h^3$$

Givens: $h = 10\,feet, \frac{dV}{dt} = ?, \frac{dh}{dt} = 5\,ft/\min$

$$V = \frac{1}{12}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{1}{12}\pi\cdot 3h^2\frac{dh}{dt} \Rightarrow \frac{dV}{dt} = \frac{1}{12}\pi\cdot 3\cdot(10\,ft)^2(5\,ft/\sec) \Rightarrow \frac{dV}{dt} = 125\pi\,ft/\min$$

8.



Altitude of plane after 1 hour: $\sin x = \frac{opp}{hyp} \Rightarrow \sin 30^0 = \frac{y}{500} \Rightarrow y = 250$

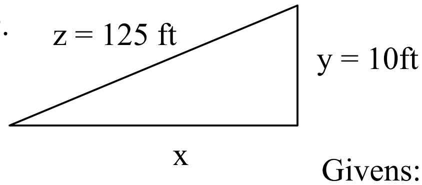
Givens: $x = 30^0, y = 250mi, z = 500mi, \frac{dz}{dt} = 500mi/hr, \frac{dx}{dt} = 0, \frac{dy}{dt} = ?$

$$\sin x = \frac{y}{z} \Rightarrow \sin x = yz^{-1} \Rightarrow \cos x\frac{dx}{dt} = \frac{dy}{dt}z^{-1} + -1z^{-2}\frac{dz}{dt}\cdot y$$

$$\cos 45^0(0) = \frac{dy}{dt}(500mi)^{-1} - (500mi)^{-2}(500mi/hr)(250mi)$$

$$0 = \frac{dy}{dt}\cdot\frac{1}{500mi} - \frac{(500mi/hr)(250mi)}{(500mi)^2} \Rightarrow \frac{dy}{dt} = 250mi/hr$$

9.



Distance the boat is from the dock:

$z^2 = x^2 + y^2 \Rightarrow 125^2 = x^2 + 10^2 \Rightarrow x = 124.6 \text{ or } 15\sqrt{69}$

Givens: $x = 15\sqrt{69}\,ft, y = 10\,ft, z = 125\,ft, \frac{dx}{dt} = 12\,ft/\min, \frac{dy}{dt} = 0, \frac{dz}{dt} = ?$

$$z^2 = x^2 + y^2 \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow (125\,ft)\frac{dz}{dt} = 15\sqrt{69}\,ft \cdot (12\,ft/\min) + 10\,ft \cdot (0)$$

$$\frac{dz}{dt} = \frac{15\sqrt{69}\,ft \cdot (12\,ft/\min)}{125\,ft} = \frac{36\sqrt{69}}{25}\,ft/\min \text{ or } 11.96\,ft/\min$$

10. Givens: $x = 1, y = 2, \frac{dx}{dt} = 6units/\sec, \frac{dy}{dt} = ?$

$$\frac{xy^3}{1+y^2} = \frac{8}{5} \Rightarrow 5\left(xy^3\right) = 8\left(1+y^2\right) \Rightarrow 5xy^3 = 8 + 8y^2$$

a) $5\frac{dx}{dt} \cdot y^3 + 3y^2\frac{dy}{dt} \cdot 5x = 16y\frac{dy}{dt} \Rightarrow 5(6units/\sec)(2)^3 + 3(2)^2 \cdot \frac{dy}{dt} \cdot 5(1) = 16(2) \cdot \frac{dy}{dt}$

$240units/\sec + 60\frac{dy}{dt} = 32\frac{dy}{dt} \Rightarrow 240units/\sec = -28\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = -\frac{60}{7}units/\sec$

b) falling because of the negative value.