Derivative Exam

A. Find the derivative of each of the following:

1. $f(x) = 9 \Rightarrow f'(x) = 0$

2. $f(x) = 5x \Rightarrow f'(x) = 5$

3. $f(x) = x^6 \Rightarrow f'(x) = 6x^5$

4. $f(x) = 2x^{\frac{5}{3}} \Rightarrow f'(x) = 2 \cdot \frac{5}{3} x^{\frac{2}{3}} = \frac{10}{3} x^{\frac{2}{3}}$

5. $f(x) = \frac{1}{x^4} = x^{-4} \Rightarrow f(x) = -4x^{-5} = \frac{-4}{x^5}$

6. $f(x) = \sqrt[5]{x^2} = x^{\frac{2}{5}} \Rightarrow f'(x) = \frac{2}{5} x^{-\frac{3}{5}} = \frac{2}{5x^{\frac{3}{5}}}$

7. $f(x) = 5^x \Rightarrow f'(x) = 5^x \ln 5$

8. $f(x) = 3^{7x} \Rightarrow f(x) = 3^{7x} \ln 3 \cdot 7$

9. $f(x) = e^{x^2} \Rightarrow f'(x) = e^{x^2} \cdot 2x$

10. $f(x) = \ln x \Rightarrow f(x) = \frac{1}{x}$

11. $f(x) = \ln\left(3x^2\right) \Rightarrow f'(x) = \frac{1}{3x^2} \cdot 6x = \frac{2}{x}$

12. $f(x) = \log_4 x \Rightarrow f'(x) = \frac{1}{x \ln 4}$

13. $f(x) = \sin\left(3x^2\right) \Rightarrow f'(x) = \cos\left(3x^2\right) \cdot 6x$

14. $f(x) = \cos x^3 \Rightarrow f'(x) = -\sin x^3 \cdot 3x^2$

15. $f(x) = \sin^3\left(x^2\right) \Rightarrow 3\sin^2\left(x^2\right)\cos\left(x^2\right) \cdot 2x$

B. Determine the first derivative:

1. $f(x) = 5x^3 - 2x^2 + 5x - 3 \Rightarrow f'(x) = 15x^2 - 4x + 5$

2.
$$f(x)=\left(\ln\left(2x^2-5\right)\right)^3 \Rightarrow$$
$$f'(x)=3\left(\ln\left(2x^2-5\right)\right)^2\cdot\frac{1}{\left(2x^2-5\right)}\cdot 4x=12x\left(\ln\left(2x^2-5\right)\right)^2\cdot\frac{1}{\left(2x^2-5\right)}$$

3. $f(x)=7^{5x^3-3x+6}\Rightarrow f'(x)=7^{5x^3-3x+6}\cdot\ln 7\cdot\left(15x^2-3\right)$

4. $f(x)=\log_6\left(6x^5-7x\right)\Rightarrow f'(x)=\frac{1}{\left(6x^5-7x\right)\ln 6}\cdot\left(30x^4-7\right)$

5. $f(x)=e^{5x-7}\Rightarrow f'(x)=e^{5x-7}\cdot 5$

6.
$$f(x)=\left(\cos\left(6x^3-2x+1\right)\right)^3 \Rightarrow$$
$$f'(x)=3\left(\cos\left(6x^3-2x+1\right)\right)^2\cdot\left(-\sin\left(6x^3-2x+1\right)\right)\cdot\left(18x^2-2\right)$$

7. $f(x)=\ln\left(\sin\left(5^{3x+2}\right)\right)\Rightarrow f'(x)=\frac{1}{\sin\left(5^{3x+2}\right)}\cdot\cos\left(5^{3x+2}\right)\cdot 5^{3x+2}\cdot\ln 5\cdot 3$

8.
$$f(x)=\ln(3x)e^{4x}\Rightarrow$$
$$f'(x)=\frac{1}{3x}\cdot 3\cdot e^{4x}+e^{4x}\cdot 4\cdot\ln(3x)=e^{4x}\left(\frac{1}{x}+4\cdot\ln(3x)\right)=e^{4x}\left(\frac{1+4x\cdot\ln(3x)}{x}\right)$$

9.
$$f(x)=\log_3 x^2\sin(5x)\Rightarrow$$
$$f'(x)=\frac{1}{\ln 3\cdot x^2}\cdot 2x\cdot\sin(5x)+\cos(5x)\cdot 5\cdot\log_3 x^2=$$
$$\frac{2}{\ln 3\cdot x}\cdot\sin(5x)+\cos(5x)\cdot 5\cdot\log_3 x^2$$

10.
$$f(x)=\left(4x^3+5\right)^{\frac{1}{3}}(3x-7)^4\Rightarrow$$
$$f'(x)=\frac{1}{3}\left(4x^3+5\right)^{-\frac{2}{3}}\cdot 12x^2\cdot(3x-7)^4+4(3x-7)^3\cdot 3\cdot\left(4x^3+5\right)^{\frac{1}{3}}=$$
$$\left(4x^3+5\right)^{-\frac{2}{3}}\cdot(3x-7)^3\left[4x^2(3x-7)+12\left(4x^3+5\right)\right]=$$
$$\frac{(3x-7)^3\left[60x^3-28x^2+60\right]}{\left(4x^3+5\right)^{-\frac{2}{3}}}$$

11.
$$f(x)=\frac{(5x-1)^3}{(7x+2)^4}=(5x-1)^3(7x+2)^{-4}\Rightarrow$$
$$f'(x)=3(5x-1)^2\cdot 5\cdot(7x+2)^{-4}+-4(7x+2)^{-5}\cdot 7\cdot(5x-1)^3=$$
$$(5x-1)^2\cdot(7x+2)^{-5}\cdot\left[15(7x+2)-28(5x-1)\right]=$$
$$\frac{(5x-1)^2(-25x+58)}{(7x+2)^5}$$

12.
$$f(x)=6^{2x}\left(\ln x+x^2\right)^{-6}\Rightarrow$$
$$f'(x)=6^{2x}\cdot\ln 6\cdot 2\cdot\left(\ln x+x^2\right)^{-6}+-6\left(\ln x+x^2\right)^{-7}\cdot\left(\frac{1}{x}+2x\right)\cdot 6^{2x}=$$
$$2\cdot 6^{2x}\cdot\left(\ln x+x^2\right)^{-7}\left[\ln 6\cdot\left(\ln x+x^2\right)-3\left(\frac{1}{x}+2x\right)\right]=$$
$$2\cdot 6^{2x}\cdot\left(\ln x+x^2\right)^{-7}\left[\ln 6\cdot\left(\ln x+x^2\right)-3\left(\frac{1+2x^2}{x}\right)\right]$$

Use the Quotient Rule:

13.
$$f(x)=\frac{\ln x^2}{\sin x}\Rightarrow f'(x)=\frac{\frac{1}{x^2}\cdot 2x\cdot\sin x-\cos x\cdot\ln x^2}{\sin^2 x}=$$
$$\frac{\frac{2}{x}\cdot\sin x-\cos x\cdot\ln x^2}{\sin^2 x}=\frac{2\cdot\sin x-x\cdot\cos x\cdot\ln x^2}{x\cdot\sin^2 x}$$

14.
$$f(x)=\frac{e^{x^3}}{(3x-1)^3}\Rightarrow f'(x)=\frac{e^{x^3}\cdot 3x^2\cdot(3x-1)^3-3\cdot(3x-1)^2\cdot 3\cdot e^{x^3}}{\left[(3x-1)^3\right]^2}=$$
$$\frac{e^{x^3}\cdot 3\cdot(3x-1)^2\left[x^2(3x-1)-3\right]}{(3x-1)^6}=\frac{e^{x^3}\cdot 3\cdot\left[3x^3-x^2-3\right]}{(3x-1)^6}$$