$z^2 = x^2 + y^2, \frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 3$, find $\frac{dz}{dt}$ when $x = 5$ and $y = 12$.

6. If $z^2 = 5^2 + 12^2 \Rightarrow z^2 = 25 + 144 \Rightarrow z = 13$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow 13 \cdot \frac{dz}{dt} = 5 \cdot 2 + 12 \cdot 3$$

C. Optimization Problems

1. A shepherd wishes to build a rectangular fenced area against the side of a barn. He has 360 feet of fencing material, and only needs to use it on three sides of the enclosure, since the wall of the barn will provide the last side. What dimensions should the shepherd choose to maximize the area of the enclosure?

   $A = lw$

   $P = L + 2w \Rightarrow 360 = L + 2w \Rightarrow 360 - 2w = L$

   $A = (360 - 2w) \cdot w \Rightarrow A = 360w - 2w^2$

   $A' = 360 - 4w \Rightarrow w = 90$

   $L = 360 - 2(90) = 180$

2. A box with a square base has no top. If 64 cm^2 of material is used, what is the maximum possible volume

   $V = x^2 \cdot y \Rightarrow x =$ dimension of base and y is height of box

   Area of base $= x^2$

   Area of 4 sides $= 4 \cdot x \cdot y$

   Total area $= x^2 + 4xy \Rightarrow 64 = x^2 + 4xy \Rightarrow \frac{64 - x^2}{4x} = y$

   $V = x^2 \cdot \frac{64 - x^2}{4x} = 16x - \frac{1}{4}x^3 \Rightarrow V' = 16 - \frac{1}{4} \cdot 3x^2$

   $0 = 16 - \frac{1}{4} \cdot 3x^2 \Rightarrow -64 = -3x^2 \Rightarrow \sqrt{\frac{64}{3}} = x$