Definite Integrals (Final Steps of Calculation have been omitted)

1. $\int_{-1}^{2}(5x+3)dx=\left.\frac{5x^2}{2}+3x\right]_{-1}^{2}=\left(\frac{5(2)^2}{2}+3(2)\right)-\left(\frac{5(-1)^2}{2}+3(-1)\right)=$

2. $\int_{1}^{4}(2x^2-3x+1)dx=\left.\frac{2x^3}{3}-\frac{3x^2}{2}+x\right]_{1}^{4}=$

$\left(\frac{2(4)^3}{3}-\frac{3(4)^2}{2}+(4)\right)-\left(\frac{2(1)^3}{3}-\frac{3(1)^2}{2}+(1)\right)=$

3. $\int_{0}^{\pi/3}(1-2\cos x)dx=x-2\sin x]_{0}^{\pi/3}=\left((\pi/3)-2\sin(\pi/3)\right)-\left((0)-2\sin(0)\right)=$

4. $\int_{1}^{3}(x-2)(x+3)dx=\int_{1}^{3}\left(x^2+x-6\right)dx=\left.\frac{x^3}{3}+\frac{x^2}{2}-6x\right]_{1}^{3}=$

$\left(\frac{(3)^3}{3}+\frac{(3)^2}{2}-6(3)\right)-\left(\frac{(1)^3}{3}+\frac{(1)^2}{2}-6(1)\right)=$

5. $\int_{\pi/4}^{\pi/3}\cos x dx=\sin x]_{\pi/4}^{\pi/3}=\sin(\pi/3)-\sin(\pi/4)=$

6.

$\int_{1}^{9}\frac{2t^2+t^2\sqrt{t}-1}{t^2}dt=\int_{1}^{9}\frac{2t^2}{t^2}+\frac{t^2\sqrt{t}}{t^2}-\frac{1}{t^2}dt=\int_{1}^{9}2+\sqrt{t}-\frac{1}{t^2}dt=$

$\int_{1}^{9}2+t^{\frac{1}{2}}-t^{-2}dt=\left.2t+\frac{2}{3}t^{\frac{3}{2}}+t^{-1}\right]_{1}^{9}=$

$\left(2(9)+\frac{2}{3}(9)^{\frac{3}{2}}+(9)^{-1}\right)-\left(2(1)+\frac{2}{3}(1)^{\frac{3}{2}}+(1)^{-1}\right)=$

7.
$$\int_1^8 (\sqrt[3]{r} + \frac{1}{\sqrt[3]{r}})dr = \int_1^8 r^{\frac{1}{3}} + r^{-\frac{1}{3}} dr = \frac{3}{4} r^{\frac{4}{3}} + \frac{3}{2} r^{\frac{2}{3}} \Big]_1^8 =$$

$$\left(\frac{3}{4}(8)^{\frac{4}{3}} + \frac{3}{2}(8)^{\frac{2}{3}}\right) - \left(\frac{3}{4}(1)^{\frac{4}{3}} + \frac{3}{2}(1)^{\frac{2}{3}}\right) =$$

$$\int_1^2 x\sqrt{x-1}dx = x\frac{2}{3}(x-1)^{\frac{3}{2}} - \int \frac{2}{3}(x-1)^{\frac{3}{2}} dx \Big]_1^2 =$$

$$\frac{2}{3}x(x-1)^{\frac{3}{2}} - \frac{2}{3}\int (x-1)^{\frac{3}{2}} dx \Big]_1^2 = \frac{2}{3}x(x-1)^{\frac{3}{2}} - \frac{4}{15}(x-1)^{\frac{5}{2}} \Big]_1^2 =$$

$$\left(\frac{2}{3}(2)((2)-1)^{\frac{3}{2}} - \frac{4}{15}((2)-1)^{\frac{5}{2}}\right) - \left(\frac{2}{3}(1)((1)-1)^{\frac{3}{2}} - \frac{4}{15}((1)-1)^{\frac{5}{2}}\right) =$$

8.
$$u = x \Rightarrow du = dx$$

$$dv = \sqrt{x-1}dx \Rightarrow v = \int \sqrt{x-1}dx = \int (x-1)^{\frac{1}{2}} dx = \int z^{\frac{1}{2}} dz = \frac{2}{3} z^{\frac{3}{2}} = \frac{2}{3}(x-1)^{\frac{3}{2}}$$

$$z = (x-1) \Rightarrow dz = dx$$

similarly $\int (x-1)^{\frac{3}{2}} dx \Rightarrow \frac{2}{5}(x-1)^{\frac{5}{2}}$

9.

$$\int_0^4 \frac{x}{\sqrt{1+2x}}dx = \int_0^4 \frac{\frac{u-1}{2}}{\sqrt{u}}\frac{1}{2}du = \int_0^4 \frac{u-1}{2u^{\frac{1}{2}}}\cdot\frac{1}{2}du = \frac{1}{4}\int_0^4 \frac{u-1}{u^{\frac{1}{2}}}du = \frac{1}{4}\int_0^4 u^{\frac{1}{2}} - u^{-\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\Bigg]_0^4 =$$

$$\frac{2u^{\frac{3}{2}}}{3} - 2u^{\frac{1}{2}}\Bigg]_0^4 = \frac{2(1+2x)^{\frac{3}{2}}}{3} - 2(1+2x)^{\frac{1}{2}}\Bigg]_0^4 =$$

$$\left[\frac{2(1+2(4))^{\frac{3}{2}}}{3} - 2(1+2(4))^{\frac{1}{2}}\right] - \left[\frac{2(1+2(0))^{\frac{3}{2}}}{3} - 2(1+2(0))^{\frac{1}{2}}\right] =$$

$$u = 1+2x \Rightarrow \frac{u-1}{2} = x$$

$$u = 1+2x \Rightarrow du = 2dx \Rightarrow \frac{1}{2}du = dx$$

10.

$$\int_0^2 \frac{x}{(x^2-1)^2}dx = \frac{1}{2}\int_0^2 \frac{1}{(u)^2}du = \frac{1}{2}\int_0^2 u^{-2}du = \frac{1}{2}\frac{u^{-1}}{-1}\Bigg]_0^2 = -\frac{1}{2u}\Bigg]_0^2 = -\frac{1}{2\left(x^2-1\right)}\Bigg]_0^2$$

$$\left[-\frac{1}{2\left((2)^2-1\right)}\right] - \left[-\frac{1}{2\left((0)^2-1\right)}\right] =$$

$$u = x^2-1 \Rightarrow du = 2xdx \Rightarrow \frac{1}{2}du = xdx$$

11. $$\int_1^4 \sqrt{x}\ln x dx = \frac{2}{3}x^{\frac{3}{2}} \cdot \ln x - \int_1^4 \frac{2}{3}x^{\frac{3}{2}} \cdot \frac{1}{x}dx = \frac{2}{3}x^{\frac{3}{2}} \cdot \ln x - \frac{2}{3}\int_1^4 x dx =$$

$$\frac{2}{3}x^{\frac{3}{2}} \cdot \ln x - \frac{2}{3}\frac{x^2}{2}\Big]_1^4 = \frac{2}{3}x^{\frac{3}{2}} \cdot \ln x - \frac{x^2}{3}\Big]_1^4 =$$

$$\left[\frac{2}{3}(4)^{\frac{3}{2}} \cdot \ln(4) - \frac{(4)^2}{3}\right] - \left[\frac{2}{3}(1)^{\frac{3}{2}} \cdot \ln(1) - \frac{(1)^2}{3}\right] =$$

$$u = \ln x \Rightarrow du = \frac{1}{x}dx$$

$$dv = x^{\frac{1}{2}}dx \Rightarrow \int x^{\frac{1}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}x^{\frac{3}{2}}$$

12. $$\int_0^1 x^2 e^{-x}\, dx$$

$$x^2 \cdot -e^{-x} - \int -e^{-x} \cdot 2x\, dx \Rightarrow -x^2 e^{-x} + 2\int xe^{-x}\, dx$$

$$u = x^2 \qquad v = -e^{-x}$$

$$du = 2xdx \qquad dv = e^{-x}dx$$

$$-x^2 e^{-x} + 2\left[x \cdot -e^{-x} - \int -e^{-x}\, dx\right] \Rightarrow -x^2 e^{-x} - 2xe^{-x} + 2\int e^{-x}\, dx$$

$$u = x \qquad v = -e^{-x}$$

$$du = dx \qquad dv = e^{-x}dx$$

$$-x^2 e^{-x} - 2xe^{-x} + 2 \cdot -e^{-x} \Rightarrow -x^2 e^{-x} - 2xe^{-x} - 2e^{-x}\Big]_0^1$$

$$v = -e^{-x}$$

$$dv = e^{-x}dx$$

$$\left(-(1)^2 e^{-(1)} - 2(1)e^{-(1)} - 2e^{-(1)}\right) - \left(-(0)^2 e^{-(0)} - 2(0)e^{-(0)} - 2e^{-(0)}\right) =$$

13.

$$\int_0^2 \frac{x^3+x^2-12x+1}{x^2+x-12}\,dx = \int_0^2 x + \frac{1}{x^2+x-12}\,dx \text{ (long division)} \Rightarrow$$

$$\int_0^2 x + \int_0^2 \frac{1}{x^2+x-12}\,dx \Rightarrow$$

$$\int_0^2 x = \frac{x^2}{2}\Bigg]_0^2 = \left(\frac{(2)^2}{2}\right) - \left(\frac{(0)^2}{2}\right) =$$

$$\int_0^2 \frac{1}{x^2+x-12}\,dx = \text{ using partial fractions}$$

$$x^2+x-12 = (x+4)(x-3) \Rightarrow \frac{A}{(x+4)} + \frac{B}{(x-3)} \Rightarrow \frac{Ax-3A+Bx+4B}{(x+4)(x-3)} \Rightarrow$$

$$(A+B)x + (-3A+4B) = 0x+1 \Rightarrow A+B=0 \text{ and } -3A+4B=1$$

By system of equatons: $A = -\frac{1}{7}, B = \frac{1}{7}$

$$\therefore \int_0^2 \frac{1}{x^2+x-12}\,dx = \int_0^2 \frac{-\frac{1}{7}}{(x+4)} + \frac{\frac{1}{7}}{(x-3)} = -\frac{1}{7}\int_0^2 \frac{1}{(x+4)} + \frac{1}{7}\int_0^2 \frac{1}{(x-3)} \Rightarrow$$

$$-\frac{1}{7}\ln(x+4)\Big]_0^2 + \frac{1}{7}\ln(x-3)\Big]_0^2 =$$

$$-\frac{1}{7}\left[\left(\ln(2+4)\right) - \left(\ln(0+4)\right)\right] + -\frac{1}{7}\left[\left(\ln(2-3)\right) - \left(\ln(0-3)\right)\right] =$$

14.

$$\int_0^1 \frac{x}{x^2+x+1}dx \Rightarrow$$

complete the trinomial square $\left(x^2+x+\frac{1}{4}\right)+1-\frac{1}{4} \Rightarrow \left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

$u = x+\frac{1}{2} \Rightarrow u-\frac{1}{2} = x$ and $du = dx$

therefore $\int_0^1 \frac{x}{x^2+x+1}dx \Rightarrow \int_0^1 \frac{u-\frac{1}{2}}{u^2+\frac{3}{4}}du \Rightarrow \int_0^1 \frac{u}{u^2+\left(\frac{\sqrt{3}}{2}\right)^2}du - \frac{1}{2}\int_0^1 \frac{1}{u^2+\left(\frac{\sqrt{3}}{2}\right)^2}du \Rightarrow$

$$\int_0^1 \frac{u}{u^2+\left(\frac{\sqrt{3}}{2}\right)^2}du \Rightarrow \int_0^1 \frac{2}{v}dv \Rightarrow 2\ln v\Big]_0^1 = 2\ln\left(u^2+\left(\frac{\sqrt{3}}{2}\right)^2\right)\Bigg]_0^1 = 2\ln\left(\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2\right)\Bigg]_0^1 =$$

$$v = u^2+\left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow dv = 2udu$$

$$2\ln\left(\left((1)+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2\right) - 2\ln\left(\left((0)+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2\right) =$$

$$-\frac{1}{2}\int_0^1 \frac{1}{u^2+\left(\frac{\sqrt{3}}{2}\right)^2}du = -\frac{1}{2}\cdot\left(\frac{1}{\frac{\sqrt{3}}{2}}\arctan\frac{u}{\frac{\sqrt{3}}{2}}\right)\Bigg]_0^1 = -\frac{1}{2}\cdot\left(\frac{1}{\frac{\sqrt{3}}{2}}\arctan\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\Bigg]_0^1 =$$

$$-\frac{1}{2}\left[\left(\frac{1}{\frac{\sqrt{3}}{2}}\arctan\frac{(1)+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) - \left(\frac{1}{\frac{\sqrt{3}}{2}}\arctan\frac{(0)+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right] =$$

$$\int_{1/3}^3 \frac{\sqrt{x}}{x^2+x}dx = \int_{1/3}^3 \frac{x^{\frac{1}{2}}}{x^2+x}dx = \int_{1/3}^3 \frac{x^{\frac{1}{2}}}{x(x+1)}dx = \int_{1/3}^3 \frac{1}{x^{\frac{1}{2}}(x+1)}dx \Rightarrow$$

$$u = x^{\frac{1}{2}} \Rightarrow u^2 = x \text{ and } du = \frac{1}{2}x^{-\frac{1}{2}}dx \Rightarrow du = \frac{1}{2x^{\frac{1}{2}}}du$$

15.

$$\frac{1}{2}\int_{1/3}^3 \frac{1}{(u^2+1)}du \Rightarrow \frac{1}{2}\left(\arctan u\Big]_{1/3}^3\right) = \frac{1}{2}\left(\arctan x^{\frac{1}{2}}\Big]_{1/3}^3\right) =$$

$$\frac{1}{2}\left[\arctan(3)^{\frac{1}{2}} - \arctan(1/3)^{\frac{1}{2}}\right] =$$